Creative Thinking

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1 Kelsie's Coin Collections

Kelsie has a coin jar full of pennies (worth 1 cent) and nickels (worth 5 cents). She likes to take different collections of her coins to see how much each collection is worth. For example, a collection of four pennies and six nickels is worth $4 \cdot 1 + 6 \cdot 5$ cents, totaling 34 cents. Every collection must have at least one penny and one nickel.

a) Determine the minimum number of coins in a collection so that the smallest possible value of a collection with that number of coins, and the largest possible value of a collection with that number of coins, differ by a positive multiple of seven cents.

b) Determine the minimum number of coins in a collection so that the number of nickels is exactly twice the number of pennies, and the total value (in cents) is a Fibonacci number (1, 1, 2, 3, 5, 8, 13, . . . , where each number is the sum of the two previous numbers).

2 Kelsie's Colored Coins and Connecting Curves

Kelsie has another jar of coins. These coins come in pretty colors: red, blue, and green. To pass the time, she pulls out coins of different colors, places them on a large sheet of paper, and draws one curve connecting each two coins that are different colors. For instance, if she were to pull out 1 red coin, 1 blue coin, and 2 green coins, she would draw five connecting curves—two from the red to each of the two green coins, two from the blue to each of the two green coins, and one more connecting the red and blue coins.

a) Kelsie pulls out 4 red, 3 blue, and 3 green coins. How many connecting curves will she draw?

b) Kelsie pulls out 20 coins. What is the maximum possible number of curves she might have to draw?

3 Roy's Restricted Robots

Roy has manufactured a large number of robots. Unfortunately these robots are restricted to moving backward and forward on a number line. Worse, each robot has a code, (f, b), where f is the number of steps it can go forward and b is the number of steps it can go backward. For instance, the robot with the code $(4, 7)$, if it starts at zero, can visit the numbers 4, 8, 12, …, or the numbers −7, −14, −21, …. Or it could go one step forward then one step back and visit the number −3, and then take another step forward to 1. Now notice that since it can get to 1, it can repeat those steps and get to 2, then 3, then …, eventually visiting all positive numbers. The $(4, 7)$ robot can also visit all negative numbers (get to 6, then take a backward trip by 7 spaces to get to −1, and once −1 has been reached, the steps can be repeated to get to −2, −3, …). So given time it can get anywhere on the number line. The robot with code $(2, 2)$ is not so lucky! Since it can only go two steps forward or two steps backward, it can only visit even numbers!

a) Robots with codes $(18, 14)$ and $(9, 33)$ both start at zero. What is the smallest positive number that they can both visit?

b) One robot, with code $(27, 15)$ starts at the origin. Another robot, with the code $(35, 49)$, starts at the number 1. What is the smallest positive number that these two robots can both visit?.

4 Andrew's Awesome Algorithm

Andrew is bored so he doodles some fractions on his paper:

$$\frac{5}{3}, \frac{5}{2}.$$

He then takes various starting numbers and plays a game:

1. He takes his starting number and multiplies it by the first fraction in his list that produces a whole number.
2. He takes that new number and does the same thing over and over.
3. If there are no fractions in the list that would produce a whole number, he stops.

For example, if he plays the game starting with 12, he next gets $\frac{5}{3} \cdot 12 = 20$. But multiplying 20 by $\frac{5}{3}$ doesn't produce a whole number, so he tries $\frac{5}{2} \cdot 20 = 50$. Next he gets $\frac{5}{2} \cdot 50 = 125$. He must stop with 125 because neither fraction in his list can be multiplied by 125 to produce a whole number.

Andrew notices something really neat! If he starts with the number $2^a \cdot 3^b$ he ends up with the number 5^{a+b}. This is because each time he multiplies by $\frac{5}{3}$ he drains one power off of the 3 and moves it onto the 5. The same thing goes for multiplying by $\frac{5}{2}$ except the power is drained off of the 2 instead of the 3. So eventually all the powers are drained off the 2 and the 3 and placed onto the 5.

Andrew tries another collection of fractions:

$$\frac{5}{6}, \frac{5}{2}, \frac{5}{3}.$$

This time, he notices that when he starts with the number $2^a \cdot 3^b$ he ends up with the number $5^{\max(a,b)}$. That is, he gets 5, raised to the power of the larger of a and b. Playing around a little more, he finds that

$$\frac{5}{6}, \frac{1}{2}, \frac{1}{3}$$

turns $2^a \cdot 3^b$ into $5^{\min(a,b)}$.

a) Assume that $a \geq b$. Using the list

$$\frac{1}{6}, \frac{5}{2}$$

the result of starting with $2^a \cdot 3^b$ is 5^x. Express x in terms of a and b (such as "$a + b$" or "$\max(a, b)$" as in the examples).

b) Find a list of fractions that starts with $2^a \cdot 3^b$ and ends with 5^{a-b} when $a \geq b$ and 7^{b-a} when $b \geq a$.