8th Grade Team

IMSA *Mu Alpha Theta*

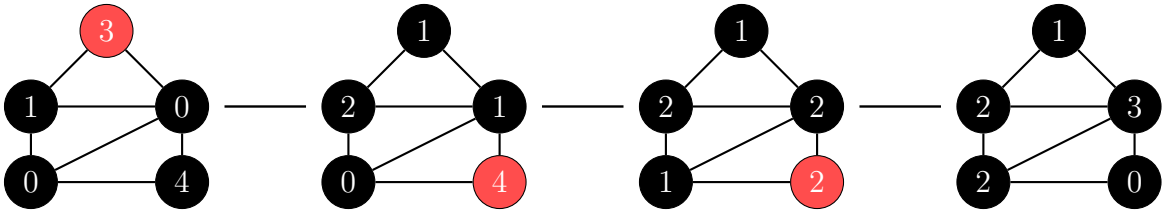
March 11, 2026

1. What is $2025^2 + 2023^2 - 2024^2 - 2022^2$?

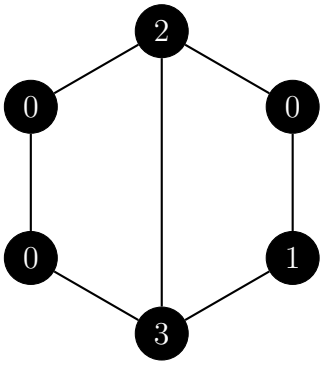
2. Five people have a combined age of 300. If the oldest person is four times the age of the youngest, what is their average age?

3. What is the largest number that divides both $100 + 123$ and $100^2 + 123^2$?

4. Roy has found out a way to clone himself multiple times, and wants to play a game with them. On a graph, each point (with some lines connecting to them) is assigned a number of Roys. If the number of Roys at that point is greater than or equal to the number of lines connecting to the point, the Roys run rampant and 1 Roy is sent along each line to the neighboring points. Here is an example game (red denotes Roys running rampant away from that point)t:



We call the final state of the Roy game *rambunctious* if no more Roys can run rampant. For the given Roy game, how many rampant runs will it take to reach a *rambunctious* Roy game?



5. A time traveler from the year 1428 traveled backwards in time. She meets three locals, the first of which says that the current year begins with a 3. The next says that the sum of the digits of the year is 12. The last says that the year is divisible by 5 but not by 10. What year has the time traveler ended up in?

6. Andrew's next test score is always a half of the sum of all of his previous tests, and there is always exactly enough extra credit to make this happen. Provided he scored a $58, 64$, and 70 on his first, second, and third tests, respectively, how many equally-weighted tests will he have taken by the time he obtains a 90 test average?

7. Roy and Andrew place 13 total rooks on two different standard 8×8 chessboards such that not a single rook attacks each other on either board. Drew notices something odd, and states that Roy's chess board contains the maximum number of empty squares not under attack, R, and Andrews board contains the minimum number of squares not under attack, A. Compute the difference $R - A$.

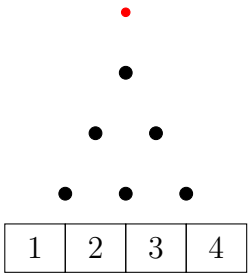
8. 97% of IMSA's class of 1997 did not fail their math class. This number is equal to 96% of the class of 1996. What is the smallest number of students that could be in the combined class of 1996 and 1997?

9. Suppose the sum of 2025 consecutive integers is $2025 \cdot 2026$. If this is true, what is the sum of the 2024th and 2023rd terms?

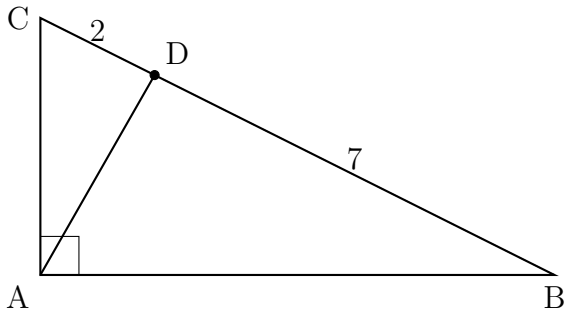
10. Pieces of candy come in masses of 3g, 7g, and 11g. Sonit is making a bag of candy and he wants the overall average of the bag to be exactly 8g per piece. What is the minimum number of pieces he needs?

11. The number of crows in a certain murder in a specific year can be modeled by the equation $4x^4 - 61x^2 + 225$, where x represents the week of the year. Suppose c is the number of crows in the murder in the 37th week of the year. What is the sum of the distinct prime factors of c?

12. At a game show, a plinko board is set up with 3 rows of pegs and 4 boxes at the bottom. You drop two balls from the top, and they bounce off the pegs, finally landing in one of the boxes. At each peg, the ball has an equal probability of going left or right. The boxes are numbered 1 through 4, from left to right. What is the probability that both balls land in box 3?



13. Drew's greenhouse is triangular. In fact, it forms a right triangle. The door to his greenhouse is on the hypotenuse of the right triangle and the door divides the hypotenuse into two segments of lengths 7 and 2. Furthermore, if Drew creates a walkway from the door directly to the opposite corner, it forms a perpendicular line to the hypotenuse. What is the total floor area of Drew's greenhouse?

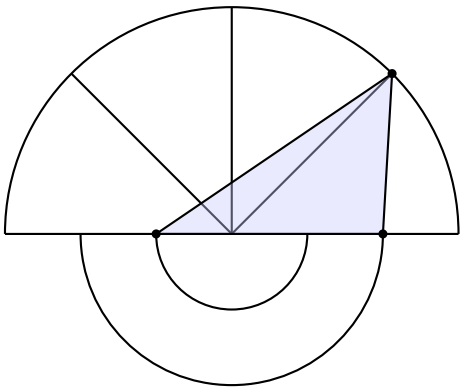


14. Let $h(x) = x^3 + 3x^2 - 10x - 23$ and $g(x) = x$. If there are three integer values for x that satisfy $g(h(x)) = 1$, and one of them is -4, find the value of the only positive integer that satisfies this condition.

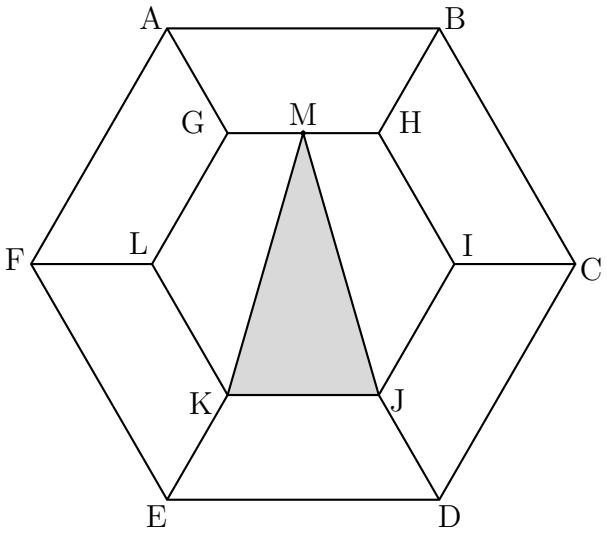
15. When $(2x^{10} + 3x^{11} + 5x^{101} + 7x^{100})(9x^{111} + 11x^{110} + 13x^{1101} + 17x^{1100})$ is fully expanded, how many terms have an even coefficient?

16. Recall that the factorial of a number n, denoted n!, is defined by $n! = n(n-1)(n-2)(n-3)\ldots 3*2*1$. Aarav starts summing up $(1*1!) + (2*2!) + (3*3!)\ldots(41*41!) = k! - m$ for some positive integers m and k. Compute $k - m$.

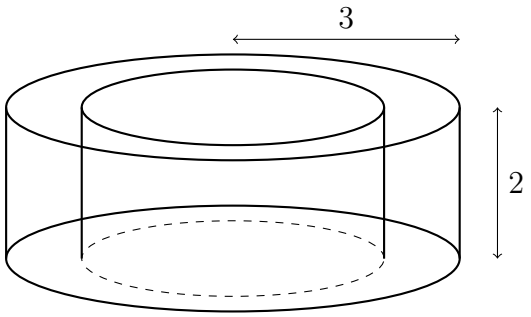
17. Three concentric semi-circles are drawn below, with radii 3, 2, 1, respectively. The largest is divided into 4 equal slices, and a triangle is drawn as shown below. What is the shaded area, expressed as a simplified radical fraction?



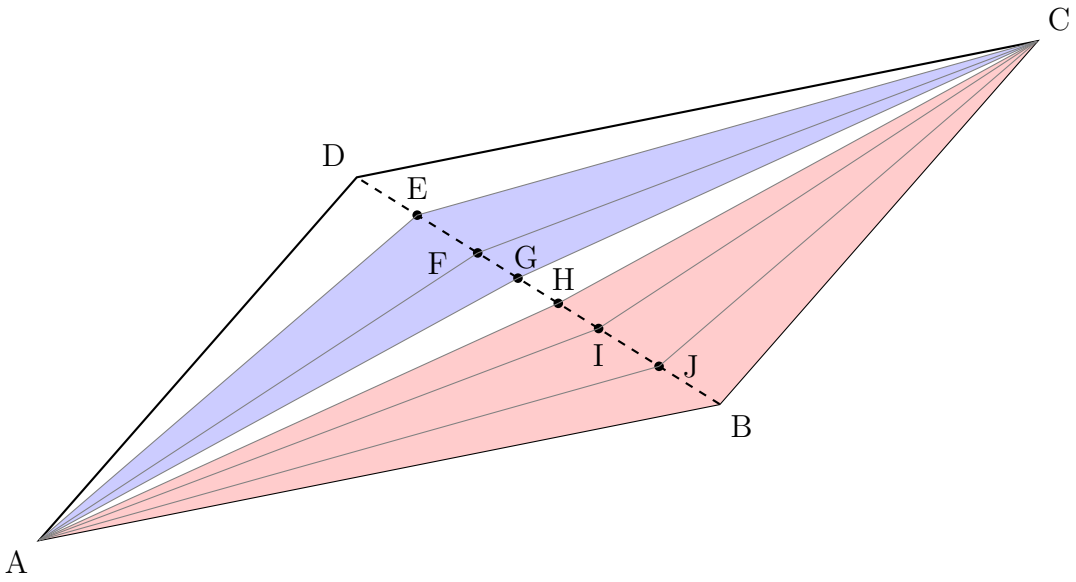
18. Ian is learning to draw eyes. In the following diagram, $ABCDEF$ and $GHIJKL$ are regular hexagons. Their vertices are connected, forming six congruent isosceles trapezoids. ΔJKM is formed by drawing M as the midpoint of $\overline{GH}$. If $\overline{AG} = 2$, $\overline{AB} = 6$, find the area of ΔJKM. Note that the sum of the interior angles of a hexagon is 720°.



19. Drew is running a donut shop. He starts with a cylindrical piece of dough with a radius of 3 inches and height of 2 inches, then randomly chooses a smaller radius, from 0 to $\frac{3}{2}$ inches, to cut a smaller cylindrical donut hole in the center. Since he doesn't want to waste the hole he cuts, he makes donut holes by cutting out the largest sphere of dough that he can fit in this smaller cylinder. What is the probability that the sphere's volume is between $\frac{9}{16}\pi$ and $\frac{4}{3}\pi$ cubic inches?



20. $ABCD$ is a parallelogram with diagonal $\overline{DB}$. $\overline{DB}$ is trisected at points F and I. Segment $\overline{FI}$ is trisected by points G and H. Lastly, segments $\overline{DG}$ and $\overline{IB}$ are bisected by points E and J, respectively. What is the ratio of the area of the blue region to the area of the red region?



21. Consider $f(x) = x^4 + ax^3 + bx^2 + cx + d$, where a, b, c, d are real coefficients. Suppose: $f(1) = 1, f(2) = 2, f(3) = 3, f(4) = 4$. Compute $f(5)$. Hint: Can you define a new function related to $f(x)$ with properties that you already know?