8th Grade Individual Solutions

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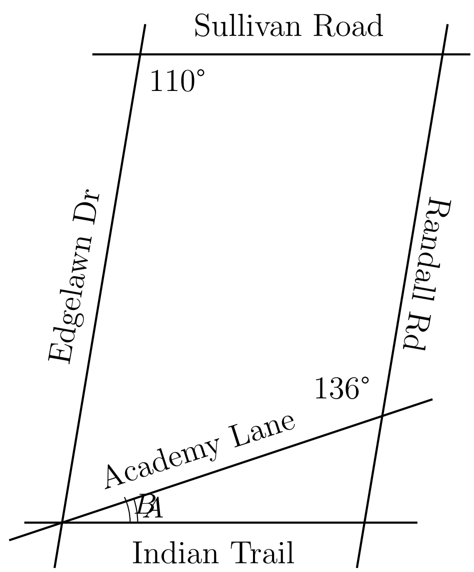
1. $7 + 12 = \boxed{19}$

2. The triangle inequality tells us that the third length of a given triangle must not exceed the sum of the other two side lengths. Hence, the maximum integer side length of the third side of the sail must be $3 + 4 - 1 = \boxed{6}$.

3. We see that Academy line is a transveral as it cuts parallel lines. $\angle A + \angle B = \frac{360-2\cdot 110}{2} = 70$. $\angle B = 180 - 136 = 44$, so $\angle A = 70 - 44 = \boxed{26}$.



4. Draw a diagram. $\angle BAD = 36 + 60 = 96°$. Also, $\angle ABC = \angle ACB = (180 - 36)/2 = 72°$. Finally, $\angle BCD = 60 + 72 = 132°$. Therefore, the greatest angle is $\boxed{132°}$.

5. Let x, y represent the side lengths of the smaller and larger gardens, respectively. We have $x + y = 13$ and $xy = 42$. It is evident that x must be 6, y must be 7; therefore, the ratio of areas is $\boxed{\dfrac{36}{49}}$.

6. Roy's polynomial is $f(x) = C(x-3)(x-5)(x-7)$. Plugging in $x = 4$ gives $C(1)(-1)(-3) = 6 \implies 3C = 6 \implies C = 2$. So $f(x) = 2(x-3)(x-5)(x-7)$., and $f(2) = 2(-1)(-3)(-5) = \boxed{-30}$

7. Use the common denominator of ab to simplify the expression as $\frac{a^2+ab+b^2}{ab} = \frac{(a+b)^2-2ab+ab}{ab} = \frac{169-17}{17} = \boxed{\dfrac{152}{17}}$.

8. Note that there are two pairs of complementary angles in the diagram, since we have a pair of 1x3 right triangles and a pair of 2x3 right triangles. The fifth triangle is the 3x3 right triangle (isosceles), so this angle is 45 degrees. Total: $45° + 90° + 90° = \boxed{225°}$.

9. The area of a triangle is $\frac{1}{2}\cdot b\cdot h$, where b and h represent the base and height of the triangle. To find the height of the isosceles triangle, draw an altitude from the base of the triangle to the opposite vertex, splitting the triangle into two congruent right triangles. Notice that for each right triangle, hypotenuse of length $70\sqrt{2}$ and base of length $\frac{112\sqrt{2}}{2} = 56\sqrt{2}$ forms a 3-4-5 right triangle with a common factor of $14\sqrt{2}$. Alternatively, one can use the Pythagorean theorem and a difference of squares to simplify the computation of $(70\sqrt{2})^2 - (56\sqrt{2})^2$. This yields a height of $42\sqrt{2}$, giving a total area of $112\sqrt{2}\cdot 42\sqrt{2}\cdot 0.5 = 56\sqrt{2}\cdot 42\sqrt{2} = 56\cdot 42\cdot 2$. If C has side length s, we know that the surface area of C is $6\cdot s^2 = 56\cdot 42\cdot 2 \implies s^2 = 56\cdot 7\cdot 2 = 7\cdot 8\cdot 7\cdot 2 = 7^2\cdot 4^2 \implies \boxed{s = 7\cdot 4 = 28}$. Note that not simplifying products makes the problem easier to solve and makes it less likely for you to make a mistake.

10. Consider $mn + 7m + 6n$. We can write this as $(m+7)(n+6) \pm c$ for some constant c. Expanding, we find that $c = 42$ so $(m+7)(n+6) - 42 = mn + 7m + 6n$. In our original equation $mn + 7m + 6n = 6$, we rewrite this as $mn + 7m + 6n + 42 = 48 \Rightarrow (m+7)(n+6) = 100$. We know that m and n are integers, so we rewrite 40 as $2^4 * 3$ and do some casework. Case 1: $m + 7 = 8, n + 6 = 6$. This leaves us with $m = 1$ and $n = 0$. This is actually the only case because any other way of splitting the factors of 48 result in one of the factors being less than 6 or 7. Our answer is $1 + 0 = \boxed{1}$.

11. $x = \frac{1}{2+\frac{1}{0+\frac{1}{2+\frac{1}{6+\frac{1}{x}}}}}$. Simplifying this recursive fraction leads to $2x^2 + 12x - 3 = 0$ Thus $a = 2, b = 12, c = -3$. Sum is $\boxed{11}$.

12. Completing the square leaves us with $(P+7)^2 < 225 \implies 3 \le P < 8$. P can be 3, 4, 5, 6, or 7. Using $\frac{P}{4} < a < \frac{P}{2}$, we find $1 + 0 + 1 + 1 + 2 = \boxed{5}$ possible magical isosceles triangles.

13. Since the sum of the first n odd numbers results in the nth perfect square, we first divide 576 by 9 to obtain 64, which is the 8th perfect square. We'd find that this is very close to the given number, so we can add 2n+1 and see that the answer is 8*9-1, or $\boxed{71}$.

14. We just need n to be equal to $60 - 3n$. For this to happen, we have $n = 60 - 3n$ and $4n = 60$, meaning Roy should give them the problem $\boxed{15}$ seconds into the tutoring session.

15. The square is maximal when the side lies on the base of the triangle. Using similar triangles, $s = \frac{bh}{b+h}$. When base $b = 6, h = 4, s = 2.4$. When base $b = 5, h = 4.8, s = \frac{120}{49}$.

$\frac{120}{49} > 2.4$, so $\boxed{\frac{120}{49}}$. 120 - 49 = $\boxed{71}$.

16. $S = 13k+1$. If S is even, $S = 2a, S^2 = 4a^2$. If a is odd, then the remainder of S^2 divided by 8 is even, as is the case when a is even. Therefore For S^2 to be odd mod 8, S must be odd, so k must be even $(2m)$. $S = 26m + 1$. We want $26m + 1 \leq 1000$. m can be $0, \ldots, 38$. Total $\boxed{39}$ integers.

17. By power of a point, we have that -16=AO*OC. But by similar triangles, we have that AE/EC = BE/OE. Coupled with power of a point, we can multiply by AE*EC to get 16. Since we know that $\frac{AE}{EC} = \frac{BE}{OE}$, we can multiply both sides by $AE \cdot EC$ to get $AE^2 = \frac{16}{3} \cdot BE$. Repeating on the other diagonal, we get $BE^2 = \frac{16}{3} \cdot AE$, and combining these two equations, we get $AE \cdot BE = \frac{256}{9}$. Then, $h = (AE + BE)^2 = AE^2 + BE^2 + 2(AE \cdot BE)$, and $h = \boxed{4\sqrt{5}}$

18. Every n-th red piece adds $\pi(n^2) - \pi(n-1)^2 = \pi(2n-1)$ more to the area. The areas of the red rings are $1\pi, 5\pi, 9\pi, 13\pi, \ldots$, so the area of the xth red ring is $\pi(4x-3)$. Thus, the sum of the rings is $\pi\left[4\frac{x(x+1)}{2} - 3x\right]$, or $\pi(2x^2 - x)$. As we need this sum to be at least 2025, we can write $2x^2 - x \geq 2025$ By estimating and checking, we get $x = 33$, and the total number of rings is $33 * 2 - 1 = \boxed{65}$

19. There are 10 points in total.The number of ways to choose 3 points is

$$\binom{10}{3} = 120.$$

To form a right triangle, we require two line segments to be perpendicular. Since the points lie on a triangular lattice, perpendicular directions only occur when one side is *horizontal* and the other is *vertical* in the underlying triangular grid. Checking each possible horizontal base, we have:

- On the bottom row, choose two adjacent points horizontally and a third point directly above them aligned vertically. This gives 3 possible right triangles.
- On the third row, a similar construction gives 2 possible right triangles.
- On the second row, this gives 1 possible right triangle.

Thus, the total number of right triangles is

$$3 + 2 + 1 = 6.$$

So, the probability of choosing a right triangle is $\frac{6}{120} = \boxed{\frac{1}{6}}$

20. The remainder of 11^n when divided by 12 is $(-1)^n$. Using this same thinking process, 5^2 has a remainder of 1, so 5^n has a remainder of 5 when n is odd, and 1 when n is even. Our remainder that we need is 4. It is evident that when n is odd, 11^n has a remainder of -1. Thus, all we need is the next odd number greater than 3, which is $\boxed{5}$.