7th Grade Team Solutions

IMSA *Mu Alpha Theta*

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1. $\sqrt{27\sqrt{9}} = \sqrt{27 \cdot 3} = \boxed{9}$.

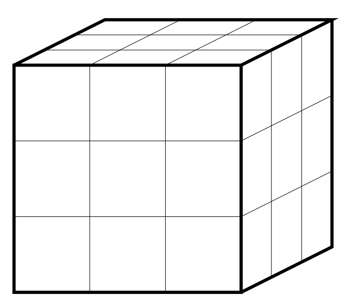
2. For each deck of cards that he obtains and then sells, it yields him a \$3 profit. So, $3x = 36 \Rightarrow x = \boxed{12}$ decks of cards.

3. The minimum number of chihuahuas is $\text{lcm}(8, 2) = 8$ (lcm is the least common multiple), so 4 are male and 7 have brown coats. However, $7+4 = 11$, so we have overcounted by 4. This overcount is the number of male brown chihuahuas since we would count them twice, once for the population of males and again for the population of brown coats. So, the answer is $\boxed{4}$.

4. If Sonit sold x cups, Eric sold $7x$ cups. Then we have $7x + x = 8x = 216 \Rightarrow x = \boxed{27}$.

5. $120\,\text{cups} = 120{\cdot}1.5$ packets used $= 180$ packets. Moreover, $180 \div 5 = 36$ packages purchased $= 36 \cdot 3$ dollars spent $= 108$ dollars spent. Finally, 120 cups sold at 1.50 is 180 dollars earned, and the total profit is $180 - 108 = \boxed{72}$.

6. Visually, looking at the cube:



   All of the unit cubes that will be painted on only 2 faces will be the edge cubes that are not corners. So, we have $\boxed{12}$ such unit cubes.

7. Tyler counts 69 numbers in 1m 55s = 115s. His rate is:

$$\frac{69}{115} = 0.6 \text{ numbers/sec}$$

   In the next 3m 15s = 195s, he counts:

$$195 \times 0.6 = 117 \text{ additional numbers}$$

   The final number is $69 + 117 = \boxed{186}$.

8. Notice the subtraction sign repeats every 3 operations. We can group the equation like this: $1+\underbrace{(2-3+4)}_{3}+\underbrace{(5-6+7)}_{6}+\underbrace{(8-9+10)}_{9}+\underbrace{(11-12+13)}_{12}+14-15$, which is easier to calculate since the first two terms of each parentheses cancels to -1. So, our equation simplifies to $1+3+6+9+12+14-15 = 3+6+9+12+15-15 = 3+6+9+12 = \boxed{30}$.

9. If he walks x minutes per mile, then in one hour he walks $60/x$ miles, so his walking speed is $60/x$ mph. His moped drives at x mph, so $x = 15\cdot(60/x) \implies x^2 = 900 \implies x = \boxed{30}$.

10. How many numbers between 25 and 100, inclusive, are divisible by either 4 or 5? The total numbers divisible by 4 or 5 in a given interval from 1 to n, inclusive, is the sum of the numbers divisible by 4 and those divisible by 5, subtracting the numbers divisible by 20 to avoid over counting the numbers divisible by both 4 and 5.
Total numbers divisible by 4 or 5 in $[1, 100]$:
$$(100/4) + (100/5) - (100/20) = 25 + 20 - 5 = 40$$
Total numbers divisible by 4 or 5 in $[1, 24]$:
$$\lfloor 24/4\rfloor + \lfloor 24/5\rfloor - \lfloor 24/20\rfloor = 6 + 4 - 1 = 9$$
Subtracting the unwanted values in the lower range:
$$40 - 9 = \boxed{31}$$

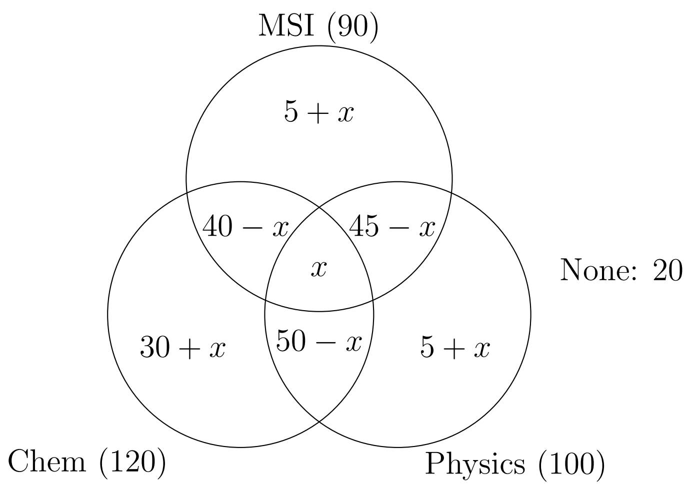
11. Consider the series as one book. There are $4! = 24$ ways to arrange the books. Then, there are another $3! = 6$ ways to arrange the books that are in a series. So, there are $24 \cdot 6 = \boxed{144}$ ways to arrange the books.

12. Only the positions of J1, J2, and Q matter. This is because however you arrange these three cards, each arrangement will have an equal number of possible permutations, as we can arrange the other cards in the same number of random ways for each arrangement. Hence, we only care about the relative ordering, as the probabilites will be preserved no matter how many other cards there are. There are $3! = 6$ arrangements, and the valid ones are J1-Q-J2 and J2-Q-J1 (2 cases). So, the probability is $2/6 = \boxed{1/3}$.

13. Let a be the number of adult tickets sold. Then the number of day passes is $a + 8$, and the number of child tickets is $a - 6$. Since the average ticket price was \$7 per customer, the total number of tickets sold is $a + (a + 8) + (a - 6) = 3a + 2$. The total amount of money spent was $5(a - 6) + 7a + 8(a + 8) = 5a - 30 + 7a + 8a + 64 = 20a + 34$. This gives us the equation $\frac{20a+34}{3a+2} = 7$, which means $7(3a + 2) = 20a + 34$. Expanding this, we get $21a + 14 = 20a + 34$, so $a = \boxed{20}$.

14. Note that the digits must alternate parity (the parity is the oddness or evenness of a number). Then, there are only two arrangements that work: Odd-Even-Odd or Even-Odd-Even. The number of ways to obtain each arrangement are OEO: $5 \cdot 5 \cdot 5 = 125$ and EOE: $4 \cdot 5 \cdot 5 = 100$ (the first digit can't be 0). Total is $125 + 100 = \boxed{225}$.

15. Let x be the number of students who took all three classes. Since 20 students took none of the classes, there are $200-20=180$ students who took at least one class. We can label the regions of a Venn diagram by subtracting the center intersection x from the pairwise overlaps: $50-x$ took Chem and Physics only, $45-x$ took Physics and MSI only, and $40-x$ took Chem and MSI only. To find those who took only one class, we subtract these overlaps from the totals: Chem-only is $120-(50-x)-(40-x)-x=30+x$, Physics-only is $100-(50-x)-(45-x)-x=5+x$, and MSI-only is $90-(40-x)-(45-x)-x=5+x$. Summing all seven regions gives $(30+x)+(5+x)+(5+x)+(50-x)+(45-x)+(40-x)+x=180$. Simplifying this, we get $175+x=180$, so $x=\boxed{5}$.



16. This is a difference of squares. Specifically, $(11^2)^2-(7^2)^2$. Using the difference-of-squares refactoring, we have $(11^2+7^2)(11^2-7^2)$. The second parentheses is another difference of squares. So we have again, $(11^2+7^2)(11+7)(11-7)=(11^2+7^2)(18)(4)=(121+49)(18)(4)=170\cdot 18\cdot 4$. 170 has one factor of 2, 18 has one factor of 2, and 4 has two factors of 2. In total, we have 4 factors of 2, or 2^4, so our answer is $\boxed{4}$.

17. If we insert a number in the beginning, ?2026, there are 9 possibilities as we can't use 0. For the second position, 2?026, we can choose any of the numbers from 0-9, which is 10 total numbers. For the remaining 3 positions, we also have 10 options. So, we have $9+10+10+10+10=49$ possibilities. However, there are repeats such as 22026, which we can get in two ways, inserting two at the first *or* second position. We see this same phenomenon where we have 2 possibilities with the same number when we insert an identical number next to a given number: 22026, 20026, 20226, 20266. Thus, there are four repeats which we must subtract giving us a final answer of $\boxed{45}$.

18. We assume that $x\leq 0$, since we want the smallest possible value, and any value greater than 0 would yield a bigger value than if we just used 0. Then, $|x|+|x+2026|=2026$ for all $|x|\leq 2026$. If $|x|>2026$, $|x|+|x+2026|>2026$ as well, so we can see that it is optimal for $|x|$ to be ≤ 2026. Then, if we choose $x=-2026$, we have $2026+|-2026+2027|=2026+1=\boxed{2027}$.

19. Note that $ab + a + b + 1 = (a+1)(b+1)$. $1 \diamond 2$ is $(1+1)(2+1) - 1 = 2 \cdot 3 - 1$. This means that $1 \diamond 2 + 1 = 2 \cdot 3 = 6$. Similarly, find that$(1 \diamond 2) \diamond 3 = 1 \cdot 2 \cdot 3 \cdot 4 - 1$. Based on the pattern we have $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 5040$.

20. Consider a checkerboard argument, tiling the entire cube like a checkerboard. Any black cube is adjacent to only white cubes, and vice versa. Thus, to end one cube adjacent to our starting position, we at least know that we have to land on a square of opposite color.

    We will now consider how to actually traverse the whole cube. One such way would be to choose the bottom $n \times n \times 1$ layer, pick a corner (for simplicity, let this be the lower left corner) and snake up by traveling all the way to the right, going one square up (in the perspective of the bottom layer), then all the way to the left, so on so forth until reaching either the top left or right corner. From here, go up to the next layer and repeat the process but backwards, leaving a column where the starting block was open. Notice that on every second iteration of "snaking up," we end up next to the column we opened up, allowing us to snake back down the column and end at an adjacent square. So, we have a successful n if n is even! So our answer is $\frac{10}{2} = \boxed{5}$.