7th Grade Individual Solutions

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1. $\boxed{8}$

2. $3 + 7 = 10$, and $10^2 = 100$. So, Ian correctly calculates the value of 100. Eric instead calculates $3^2 + 7^2 = 9 + 49 = 58$. The difference between 100 and 58 is $100 - 58 = \boxed{42}$.

3. The hour hand will it point at it twice, while the minute hand will point at it 24 hours times. So, $2 + 24 = \boxed{26}$. They won't both point at it at the same time, since whenever the hour hand is pointed directly at a number the minute hand will be pointed at 12.

4. 7 days ago it was Tuesday, 14 days ago it was Tuesday, 21 days ago it was Tuesday, 22 Monday , 23 Sunday, 24 Saturday, 25 Friday, 26, $\boxed{\text{Thursday}}$.

5. If $\frac{4}{7}$ of the balls are old, doubling the amount of balls will mean that $\frac{4}{14}$, or $\frac{2}{7}$ of the balls will be old, since the number of old balls will stay the same. Using complimentary counting, we can see that $\boxed{\frac{5}{7}}$ of all the soccer balls will be new.

6. Using intuition, the number that is equidistant from 3 and6 is 4.5 so $\boxed{2x = 9}$. Theoretically one could also use casework, with $x - 3 = -x + 6$ (yields $2x = 9$), $x - 3 = x - 6$ (has no valid solutions), $-x + 3 = -x + 6$ (has no valid solutions), $-x + 3 = x - 6$ (yields $2x = 9$).

7. The performers are not performing for $15 + 5 = 20$ total minutes. If the show starts at 7:00 pm and ends at 8:45 pm, there was a total elapsed time of $60 + 45 = 105$ minutes. $105 - 20 = \boxed{85}$.

8. Square both sides and use our formula to get $(x + \frac{1}{x})^2 = x^2 + 2(x)(\frac{1}{x}) + \frac{1}{x^2} = x^2 + 2 + \frac{1}{x^2} = 10$. Then, $x^2 + \frac{1}{x^2} = 8$. We apply the same process, yielding $(x^2 + \frac{1}{x^2})^2 = x^4 + 2(x^2)(\frac{1}{x^2}) + \frac{1}{x^4} = 64$. This gives us that $x^4 + \frac{1}{x^4} = 64 - 2 = \boxed{62}$.

9. Properties are 2 and 3 are equivalent since a number is divisible by 9 when the sum of its digits is also divisible by 9. The smallest 3-digit number that is divisible by 9 is $99 + 9 = 108$, and this happens to have all different digits, the smallest such number. The next two multiples of 9 are 117 and 126. Since $\boxed{126}$ has all unique digits, it is the second-smallest 3-digit multiple of 9 with distinct digits.
   Note on the divisibility rule for 9: This is true as any number can be decomposed

as the sum of each digit with some coefficient of 10^n. For instance, a 3-digit number can be written as $100A + 10B + C$ if its digits are A, B, C, in that order. We can rewrite it as $(99 + 1)A + (9 + 1)B + C$. This is expanded as

$$\underbrace{(99A + 9B)}_{\text{divisible by 9}} + \underbrace{(A + B + C)}_{\text{sum of digits}}$$

So, the divisibility of this expression depends on the divisibility of the sum of the digits. Clearly, this applies to any digit number, as each element of this expanded sum can be refactored into the part divisible by 9 and one additional part from each digit.

10. $\frac{a+b}{2} = 3 \implies a + b = 6$. Also, we know that $\frac{ab+b^2}{2} = 9 \implies ab + b^2 = 18$. We can find that $(a + b)^2 = a^2 + b^2 + 2ab = 6^2 = 36$. Notice that we can subtract $ab + b^2$ from $(a + b)^2 = a^2 + b^2 + 2ab$ to get $a^2 + b^2 + 2ab - ab - b^2 = a^2 + ab = 36 - 18 = 18$. Then, the average of a^2 and ab is simply $\frac{a^2+ab}{2} = \boxed{9}$.

11. For 16, two valid pairs of numbers exist, $(1, 16)$ and $(2, 8)$. For 225, we have $(1, 255)$, $(3, 75)$, $(5, 45)$, and $(9, 25)$. Of these only $(2, 8)$ and $(9, 25)$ work, so we have $2+8+9+25 = \boxed{44}$

12. The sequence goes:

$$odd, even, odd + even = odd, odd + even = odd, odd + odd = even, ...$$

More simply:

$$odd, even, odd, odd, even, odd, odd, even, odd, ...$$

As we can see, after every three terms, there is a pattern *odd, even, odd*. So for every three terms, two are odd. The largest multiple of three less than 50 is 48, which is $3 \cdot 16$, which contributes $2 \cdot 16 = 32$ odd terms. The 49th and 50th terms are *odd, even*, so $32 + 1 = \boxed{33}$ odd terms are in the first 50 terms.

13. There are 3 cases: $\mathcal{X}$ is less than the media, $\mathcal{X}$ is greater than the median, and $\mathcal{X}$ is equal to the median.

    1. If $\mathcal{X} < 5$, then the sequence is $\mathcal{X}, 3, 5, 9, 20$. The median is 5 and the mean is $\frac{\mathcal{X}+3+5+9+20}{5} = \frac{\mathcal{X}+37}{5}$. Setting the mean equal to 3 times the median gives:

    $$\frac{\mathcal{X} + 37}{5} = 15 \implies \mathcal{X} + 37 = 75 \implies \mathcal{X} = 38.$$

    However, this contradicts our assumption that $\mathcal{X} < 5$.

    2. If $5 < \mathcal{X} < 9$, then the sequence is $3, 5, \mathcal{X}, 9, 20$. The median is $\mathcal{X}$ and the mean is $\frac{3+5+\mathcal{X}+9+20}{5} = \frac{\mathcal{X}+37}{5}$. Setting the mean equal to 3 times the median gives:

    $$\frac{\mathcal{X} + 37}{5} = 3\mathcal{X}.$$

    Cross-multiplying gives:

    $$\mathcal{X} + 37 = 15\mathcal{X}.$$

Rearranging gives:

$$14\mathcal{X} = 37 \implies \mathcal{X} = \frac{37}{4}.$$

However, this contradicts our assumption that $5 < \mathcal{X} < 9$.

3. If $\mathcal{X} \geq 9$, then the sequence is $3, 5, 9, \mathcal{X}, 20$ or $3, 5, 9, 20, \mathcal{X}$. The median is 9 and the mean is $\frac{3+5+9+\mathcal{X}+20}{5} = \frac{\mathcal{X}+37}{5}$. Setting the mean equal to 3 times the median gives:

$$\frac{\mathcal{X}+37}{5} = 27 \implies \mathcal{X} + 37 = 135 \implies \mathcal{X} = 98.$$

Thus, the number inserted is $\boxed{98}$.

14. Let's instead consider each number from 1 to 11, and divide that number by the previous factorial. Notice that when our current number is prime, it won't be able to divide by the previous factorial. So, the numbers $2, 3, 5, 7, 11$ are will return non-whole numbers. This leaves $1, 4, 6, 8, 9, 10$. Of these, the only numbers which cannot be divided by their previous factorial is 4. Therefore, only $\boxed{\frac{5}{11}}$ of the numbers will return a whole number when divided by their previous factorial.

15. The maximum number of days is $12/2 = 6$ days, and the minimum is $12/4 = 3$ days. So, we only care about how many ways he can finish in 3 or 4 days.
Case 1 (3 days): $4 + 4 + 4$ is the only possibility, so we have 1 case.
Case 2 (4 days): First consider the maximum he can eat in 4 days, which is $4 \cdot 4 = 16$. Since this is 4 more fruit snacks than the box contains, we can remove 4 fruit snacks total across the four days. This way we "count backwards" from the initial 4 fruit snacks that each day has. The maximum we can take from one day is 2 fruit snacks ($4 - 2 = 2$, the minimum that he eats each day).
All the unordered possibilities are $(2, 2, 4, 4), (2, 3, 3, 4), (3, 3, 3, 3)$. The first arrangement has 6 ways of ordering, the second has 12, and the third has 1, for a total of $6+12+1 = 19$ cases. So in total we have 20 cases, or $\boxed{\text{20 different ways}}$.

16. Aarav starts at $t = 0$, so by the start of $t = 5$, he has done 20 problems. Roy starts when $t = 5$ begins. By the end of $t = 5$, or at the start of $t = 6$, he has solved $2(5) + 1 = 11$ problems, and Aarav has solved 24. Similarly, at the end of $t = 6$, or at the start of $t = 7$, Roy solved $11 + 13 = 24$ problems and Aarav has solved 28. By the end of $t = 7$, Roy has solved an additional $2(7) + 1 = 15$ problems for a total of $11 + 13 + 15 = 3 \cdot 13 = 39$ problems compared to Aarav's 32. The end of $t = 7$ corresponds with the end of the $\boxed{\text{8th second}}$.

17. The number of factors of 10 can be counted by counting the numbers of 2's and 5's. Note that 5's are less abundant than 2's; that is, a given factorial will have more factors of 2's than 5's, so we can just count how many factors of 5's any given factorial has to count its trailing zeroes. The factorization of 22! consists of the numbers from 1 to 22. Of these, only $5, 10, 15, 20$ have a factor of 5. So, in total there are $\boxed{\text{4 trailing zeroes}}$. Note

that we count one factor of five for each of the four numbers, but if one were divisible by 25, we would have to count an additional factor of 5, etc.

18. Suppose the four digits of the number are A, B, C, D, in that order from left to right. Then, the number is equal to

$$1000A + 100B + 10C + D = 9(1000D + 100C + 10B + A).$$

These values are constrained to be four-digit numbers. Specifically:

$$1000A + 100B + 10C + D \leq 9999$$

and

$$9(1000D + 100C + 10B + A) \leq 9999$$

The second inequality simplifies to:

$$1000D + 100C + 10B + A \leq 1111$$

Thus, $D = 1$ as any larger value will make this inequality invalid (and D cannot be zero, otherwise it will not form a four digit number).
Look back at the original expression. Because $D = 1$, we know that the last digit of the right side of our equation must be 1 as well. Only $9 \cdot A$ contributes to the last digit, as the other factors have a factor of ten and don't contribute to the last digit. So, $9 \cdot A$ has a last digit of 1. This is only possible when $A = 9$.
Plugging in $D = 1$ and $A = 9$ to the original equation, we get $9000 + 100B + 10C + 1 = 9000 + 900C + 90B + 81$, which becomes $10B - 80 = 890C \implies B = 89C + 8$. Clearly, $C = 0$, otherwise B is not a digit. Then, $B = 8$, and our two numbers are 9801 and $\boxed{1089}$.

19. In order to count the matches, we can see that for each of the n teams, they play $n - 1$ different teams, yielding $n(n - 1)$ games.

However, this also considers that each team is played twice (since we are considering that each team plays EVERY team no matter what), so we divide by 2 to account for any double counting to get $\frac{n(n-1)}{2}$ games.

Plugging $n = 12$ into the formula, we get $\frac{(11)(12)}{2} = 66$ games. We know that the amount of wins are the same as losses, but losses provide no total score to count, so we can ignore them, and each tie contributes a total of 2 points to count. Let w be the amount of wins and t be the amount of ties. Then,

$$w + t = 66$$

$$3w + 2t = 176$$

$$\Rightarrow w = 44, \boxed{t = 22}$$

20. Because the problem does not ask for time taken, time is irrelevant and we can ignore the minutes in which Roy thinks. Therefore, we consider the probability that he writes 130 words 5 times without giving up. This is just a $\frac{1}{2}$ chance five times, or $\frac{1}{2^5} = \boxed{\frac{1}{32}}$