8^{th} Grade Team Contest

IMSA *Mu Alpha Theta*

March 5, 2025‘

1. Compute $1/(1/(1/(1/(1/(1/2)))))$.
2. Determine the smallest positive integer that can be multiplied by 75 to become a perfect cube.
3. Mr. Michael writes the integers 1-32 on the board. He then asks each of his 31 students to come to the board in turn. When a student comes to the board they choose any two of the number, erase them, and write the number that is one less than the sum of the two numbers they just erased. When the thirty-first student does this, there will remain only one number will be written on the board. What is that number?
4. Determine the greatest integer n such that all positive integers less than $\sqrt{n}$ are factors of n.
5. Find the sum of all two-digit positive integers ab such that $a + b$ is prime and $a \cdot b$ is a perfect cube.
6. Max if M years old. Interestingly, M is the sum of the ages of his six children. X years ago, Max was twice the sum of the ages of his children. Compute Y/X.
7. Will starts with a 10×10 square in the plane aligned with the north-south and east-west directions. He draws the diagonal from the southwest corner to the northeast corner, and then draws the square that has one side along this diagonal and extends in the northwest direction. He then draws the north-south diagonal of this new square, and creates yet a larger square using this diagonal as a side, extending in the westward direction. He does this procedure (drawing the diagonal rotated 45 degrees counterclockwise from the previously drawn one, and creating a new larger square projecting in the counterclockwise direction) two more times. Will then colors in all the squares. Compute the area of the colored region.
8. The number $91a82b16$, where a and b are replaced by single digits, is divisible by 72. What is the greatest possible value of $(a + 2b)^2$?
9. Determine the number of different ordered quadruples (a, b, c, d) of positive integers that satisfy the requirement that $a + b + c + d = 10$.
10. Determine the units digit of $2021^{2025} + 2022^{2025} + 2023^{2025} + 2024^{2025}$.

11. Given that $(x+y+1)^2 = 54$ and that $(x+1)(y+1) = 15$, compute the value of $x^2 + y^2$.

12. Take any three-digit number whose digits are not all the same. Sort the digits into decreasing order and into increasing order, and subtract. Repeat the process at most seven times, and you always end up with 495! (If the subtraction ends up with a number with fewer than three digits, include enough zeroes to get it back up to three digits.) For example, start with 283. $823 - 238 = 585$, $855 - 558 = 297$, $972 - 279 = 693$, $963 - 369 = 594$, and $954 - 459 = 495$ which has the same three digits, so that pattern stays stuck on 495 forever. What number do you end up with if you do this with four-digit numbers?

13. Consider 8 dots arranged into a 2×4 square grid. How many different triangles are there with all three vertices as points in this grid?

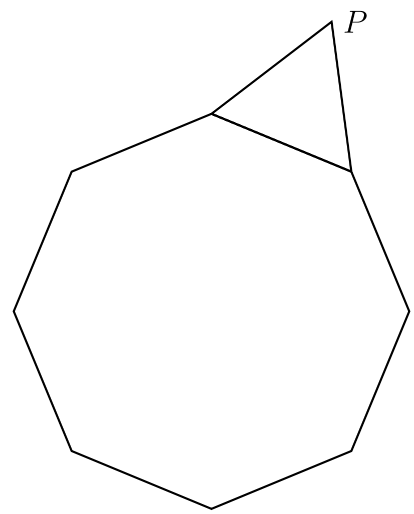
14. A number is called *semiprime* if it is the product of exactly two prime number (which can be the same). So 4, 6, 9, and 10 are all semiprime. Compute the 16^{th} semiprime number.

15. Michael has 172 identical isosceles right triangles. He wishes to glue them together to make squares of various sizes. He wants to make as many squares as possible, each a different size than the others. How many squares will he end up making?

16. How many diagonals are there in a regular 100-sided polygon?

17. Two spheres, both with radius r, are placed in space so that their centers are 32 units apart. The circle which is the intersection of the two spheres can have a rectangle inscribed in it that has dimensions 36×48. Compute the common radius, r.

18. An equilateral triangle with sides of length 5 is placed on the edge of a regular octagon whose sides also have length 5. Let P be the vertex of the triangle that is initially not touching the octagon. The triangle is rolled around the octagon (that is, it is rotated about one of the vertices that is touching the octagon until it lies against the next edge of the octagon, then rotated around the next vertex that is touching the octagon, and so on) one complete time. Compute the distance that the point P travels during one complete circuit of the triangle around the octagon.



19. Roy has 8 checkers and a regular 8×8 checkerboard. He is arranging the checkers on the board according to the following rules. The first checker he places must go in the upper-left corner of the board. After that he may place as many more checkers as he wants as long as each new checker has another checker above it or is in the top row, and each new checker has another checker to its left or is in the left-most column. In how many ways can he place between 1 and 8 checkers on the board?

20. Determine the exact value of $\dfrac{5}{5+\frac{5}{5+\frac{5}{5+\frac{5}{\cdots}}}}$.