8^{th} Grade Individual Contest

IMSA *Mu Alpha Theta*

March 5, 2025

1. How many positive factors of 561 are not prime?

2. After taking 5 tests, each worth the same amount, Karthik has an average score of 89. He needs an average score of 90 to get an A in the class. What is the minimum score he can get on his sixth test to earn an A?

3. In how many different ways could one digit be placed into each blank so that _6_8 will be divisible by 11?

4. In the number 4828 each digit is a power of two. What is the next number after 4828 for which this is true?

5. The number $2^{2025} - 2^{2022}$ can be expressed as $m \cdot 2^n$ where m is an odd number. Compute $m + n$.

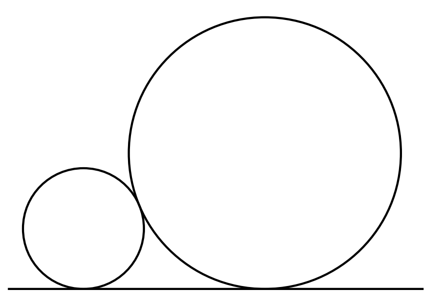
6. One hundred alarms all go off at once. The first alarm goes off every minute, the second alarm goes off every two minutes, the third alarm every three minutes, and so on. After how many minutes will at least 16 of the alarms go off at the same time again?

7. Michael has a large number of weights that weigh 3 pounds, 8 pounds, and 11 pounds. He can combine these weights to make other weights. For example, he can take three 3-pound weights and one 11-pound weight to make 20 pounds. What is the largest weight that he *cannot* make?

8. Determine the distance from the point $(6, 9)$ to the line whose equation is $5x + 12y = -8$.

9. If n is a positive integer, $n!$ means $1 \cdot 2 \cdot 3 \cdots n$. For example, $5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$. Determine the number of prime factors of $\sqrt{\dfrac{2023! + 2024!}{2023!}}$.

10. Two circles are tangent to each other and to a common tangent line. The radii of both circles and the length of the tangent are all whole numbers. One circle has radius 5 more than the other. Compute the difference between the areas of the circles.



11. For positive numbers, let $\lfloor x \rfloor$ mean the whole part of x. So $\lfloor 1.3 \rfloor = 1$, $\lfloor 1/2 \rfloor = 0$ and $\lfloor 8 \rfloor = 8$. How many solutions are there to the equation $\lfloor x \rfloor = \frac{4}{5}x + \frac{1}{2}$?

12. Will has a giant zucchini that weighs 2 pounds, but it is 95% water. He leaves it out in the sun for a couple days and some of the water evaporates, so now the zucchini is only 80% water. How much does it weight now?

13. Six slips of paper lie on the table. Each slip has a letter printed on one side and a number printed on the other. On the face-up sides of the slips are A, B, E, 3, 4, and 5. Determine how many slips must be turned over to determine whether or not the following statement is true: "if a slip has a vowel on one side, then it has a prime number on the other side."

14. Roy takes $5^3 = 125$ identical small cubes and assembles them into one $5 \times 5 \times 5$ cube. He then paints the outside of this cube before disassembling it back into the small cubes. How many of the small cubes are painted on exactly two faces?

15. The quadrilateral whose sides are the x-axis, the y-axis, the line $y = 6$, and the line $y = -2x + 10$ is rotated around the y-axis to form a three-dimensional solid object. The volume of this object can be written as $a\pi/3$ where a is a whole number. Compute the value of a.

16. Isosceles triangle ABC has sides of lengths $AB = 10$, $AC = 10$ and $BC = 12$. The reflection of point B over point A is D, and the reflection of B over C is called E. Compute the area of quadrilateral $ACED$.

17. Each corner of a tetrahedron is sliced off, leaving a triangular face in its place. Each face of the original tetrahedron has been turned into a hexagon. How many edges will this new shape have?

18. The *digisquare* of a number is the sum of the squares of its digits. For instance, the digisquare of 47 is $4^2 + 7^2 = 65$. If you keep taking the digisquare of a number and it ends up at 1, the number is called *prominent*. For example, 31 is prominent, because $3^2 + 1^2 = 10$ and then $1^2 + 0^2 = 1$, and then you get stuck at 1. Compute the largest prominent number that is less than 100.

19. Three vertices are chosen at random from a regular octagon. Compute the probability that the triangle formed from these three points is a right triangle.

20. Max and Michael are playing a game. In each round, they roll a fair die, and Max wins a point if it comes up 1 or 2; otherwise Michael wins a point. The game is currently tied, and one of the players wins if he gets two points ahead of his opponent. They continue to play until one of the players wins. What is the probability that Michael eventually wins the game?