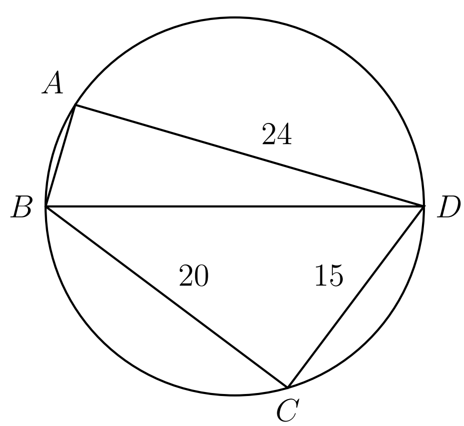
7^{th} Grade Team Contest

IMSA *Mu Alpha Theta*

March 5, 2025

1. Determine the smallest positive integer that when multiplied by 75, the result is a perfect square.
2. The number 11^7 is multiplied out. Which digit(s) appear exactly one time in the result?
3. Grant drives a total of 20 miles. For the first 10 miles, he drives slowly at a speed of 10 mile per hour. For the second 10 miles, he speeds up to 30 miles per hour. What is Grant's average speed for this trip?
4. Consider the operation $a \star b = \frac{1}{a+b}$. Compute $1 \star 1 \star 1 \star \cdots \star 1$, where there are ten $\star$'s in the expression. Normal rules of operation order apply.
5. Rose bakes a cake that is 13 inches wide and 17 inches long. How many 3-inch square servings can be cut from this cake?
6. My puppy has nine toys. Each day she selects toys to play with. Some days she doesn't play with her toys at all, other days she might play with all of them. How many different selections could she make?
7. Each day, Alia is paid \$1.50 times the calendar date. That is, on the first day of the month she is paid \$1.50. On the second day, \$3.00, and so on. On which date of the month will Alia have earned at least \$100 in total?
8. In the figure shown below, $\overline{BD}$ is a diameter of the circle. Triangles ABD and BCD are inscribed in the circle. Given that $AD = 24$, $BC = 20$, and $CD = 15$, compute the length AB.



9. Consider an imaginary number called i that obeys all the rules of arithmetic, and additionally $i^2 = -1$. Compute $(2+i)(3+4i)(5+6i)(7+2i)$.

10. Consider the system of equations

$$\begin{aligned} 12 &= a \\ 33 &= 2a + 3b \\ 57 &= 4a + 5b + 6c \end{aligned}$$

Determine the exact value of $a + b + c$.

11. Given that $(x+y+1)^2 = 54$ and that $(x+1)(y+1) = 15$, compute the value of $x^2 + y^2$.

12. Angela is a stamp collector. In her wallet, she has two \$100 bills, seven \$20 bills, four \$5 bills, and a lot of \$1 bills. At the collector store, she sees a very rare stamp that she would like to purchase. She notices that the price is the least amount possible that is more than \$150, where the digits of the price are increasing from left-to-right, and that requires the fewest number of bills from her wallet. How much did the stamp cost?

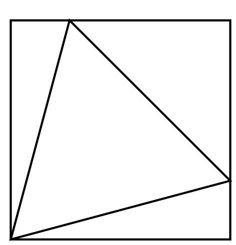
13. A group of people are sitting around a circular table. One person takes out a bag that contains 400 pieces of candy, and takes one piece, then hands the bag to the person at their right. That person takes two pieces, and hands it to the right, and the next person takes three pieces, and so on, each person taking one more piece of candy out of the bag. When the bag makes it back to the first person there is still candy in the bag but not enough to take one more piece than the previous person. How many people were at the table?

14. The height of a square pyramid is 4 meters and its total area (including the base) is $(16 + 16\sqrt{5})$ sq. meters. Compute the volume of the pyramid.

15. Michael has 172 identical isosceles right triangles. He wishes to glue them together to make squares of various sizes. He wants to make as many squares as possible, each a different size than the others. How many squares will he end up making?

16. The numbers a, b, and c are the values 5, 10, and 15 in some order. Let $x \oplus y = x + y$ and $x \otimes y = \frac{xy}{x+y}$. The expression $a \otimes b \oplus c$ is an integer. Compute $\frac{ab}{c}$.

17. The largest equilateral triangle that can fit inside a square has one of its vertices in the corner of the square and its other two vertices arranged symmetrically on the two opposite sides of the square. If the square has sides of length 2, compute the area of the triangle.



18. If a is a positive integer, then $c(a)$ means to write a twice. That is, $c(4) = 44$ and $c(123) = 123123$. Compute the sum of all positive integers $x < 100$ for which $c(x) \cdot (7) = c(y)$ for some integer y.

19. There is a unique positive integer x so that $x^6 = y^2 + 127$ for some integer y. Compute x.

20. An equilateral triangle with sides of length 5 is placed on the edge of a regular octagon whose sides also have length 5. Let P be the vertex of the triangle that is initially not touching the octagon. The triangle is rolled around the octagon (that is, it is rotated about one of the vertices that is touching the octagon until it lies against the next edge of the octagon, then rotated around the next vertex that is touching the octagon, and so on) one complete time. Compute the distance that the point P travels during one complete circuit of the triangle around the octagon.