

# 7<sup>th</sup> Grade Individual Contest

IMSA *Mu Alpha Theta*

March 5, 2025

1. Compute  $3 \cdot (1 + 5) - 7 \cdot 2 + 9$ .
2. What is 22% of 300?
3. The perimeter of a rectangle is 22. One side has length 5. Find the area of the rectangle.
4. Compute the square root of the sum of the first ten positive odd integers.
5. The operation  $\odot(n)$  is defined to mean  $\frac{n^2 - 100}{n + 10}$ . Compute the value of  $\odot(95)$ .
6. Three different numbers are chosen from the set  $\{-18, -12, -4, 3, \pi, 5, 6, 13, 16\}$ . What is the largest possible product?
7. Let  $\lfloor x \rfloor$  be the greatest integer that is not larger than  $x$ . So  $\lfloor 2.5 \rfloor = 2$ ,  $\lfloor 2 \rfloor = 2$ , and  $\lfloor -2.5 \rfloor = -3$ . Compute  $\lfloor \sqrt{1} \rfloor + \lfloor \sqrt{2} \rfloor + \lfloor \sqrt{3} \rfloor + \cdots + \lfloor \sqrt{36} \rfloor$ .
8. If  $X$  represents a single digit 0–9, determine the value of  $X$  so that 5924X1 is divisible by 11.
9. Matt has 32 plates. On the first plate is a single grain of rice. The second plate holds 2 grains of rice, the third plate has 4 grains, and each subsequent plate continues to have twice the amount of rice as the plate before it. If the number of grains of rice is totaled, what is the units digit?
10. A 10-foot tall pole is shaped like a regular hexagon. A vine begins growing around the pole so that as it grows upward it also wraps itself around the pole, growing at a  $30^\circ$  angle to the horizontal. When the vine reaches the top of the pole, compute the exact total length of the vine.
11. Will and Max are playing a game. Will starts with 7 pennies, and Max starts with 9 pennies. In each round, each player can either add one penny to their pile or they can double the number pennies in their pile. If they cooperate to finish as quickly as possible, how many round will they need so that both players have the same number of pennies in their pile?
12. Michael has a large number of three different kinds of weights: they weigh 3 pounds, 8 pounds, and 11 pounds. Michael can combine his weights to make larger weights. For instance, he can take three 3-pound weights and an 8-pound weight to make 17 pounds. What is the largest weight that Michael *cannot* make?
13. The numbers 3, 4, 5, 8, 12, and 13 are written in a line in such a way that the sum of any two adjacent numbers is a prime number. What is the sum of the first and last numbers in the line?

14. Drew is playing with two-digit integers,  $ab$  where  $a$  and  $b$  are the digits. If  $a \geq b$  then Drew's next number is  $a^2 - b^2$ , while if  $a < b$  then her next number is  $b^2 - a^2$ . She continues the process until she ends up with a single digit number (which could be zero). If she does end with zero, the original number  $ab$  is called *sneaky*. Compute the largest sneaky number less than 85.
15. An *alpha number* is a positive integer that has more distinct factors than any smaller positive integer. The first alpha number is 1 (with one factor), then 2 (with two factors), then 4 (with three factors), then 6 (with four factors). The fifth alpha number is 12 (with 6 factors). Compute the eighth alpha number.
16. If  $n$  is a nonnegative integer,  $n!$  (pronounced " $n$  factorial") means to multiply all the numbers up to  $n$ . For instance,  $4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$ .  $0! = 1$  and  $1! = 1$  by tradition. A number is called a *factorion* if it is equal to the sum of the factorials of its digits. For instance  $40585 = 4! + 0! + 5! + 8! + 5! = 24 + 1 + 120 + 40320 + 120$  is known to be the largest factorion. There is exactly one factorion with three digits. Find it!
17. Consider the sequence 1, 4, 13, 40, 121,  $\dots$ . Compute the sum of the next two numbers in this sequence.
18. A *magic square* is an arrangement of numbers into a square array so that every row, column, and diagonal have the same sum. In the example shown at right, the *magic sum*—the number to which each of the lines add—is 15. A *magic cube* is a three-dimensional arrangement of numbers so that all the rows, horizontal columns, vertical columns, and the four diagonals between opposite corners of the cube all add to the same sum. If the numbers 1–27 are arranged into a magic cube (which is possible), determine the magic sum.
- |   |   |   |
|---|---|---|
| 2 | 7 | 6 |
| 9 | 5 | 1 |
| 4 | 3 | 8 |
19. Let  $x$  be a positive integer so that  $x^2 + 13x$  is a perfect square. Compute  $x$ .
20. Libby has a balance scale that compares the weights of the objects in its two pans. Libby flips a fair coin, and if it comes up heads she places a marble into the left pan of her scale. If it comes up tails she places the marble in the right pan. If there are ever two more marbles in one pan than in the other, the scale will indicate that the pans are out of balance (it is not sensitive enough to detect just a one-marble difference). Compute the probability that Libby will place a total of five marbles without the scale going out of balance.