Creative Thinking

IMSA *Mu Alpha Theta*

February 22, 2023

1 Pie!

Shiqi has six friends who love pie. Their names are Arjun, Lily, Max, Rohan, Ryan, and Vidyoot. When Shiqi has pie, they all gather around her to ask her for some. But sometimes they don't all behave well and then Shiqi doesn't give them any pie.

a) Sunday, Shiqi had three identical pies. She gave one to each of the first three of her friends who came and asked for one. Since it didn't matter who got which pie, how many different ways were there for Shiqi to choose three of her friends and give one pie to each of the three?

b) Monday, Shiqi had three more identical pies. However, that day Vidyoot had borrowed Shiqi's viola and didn't put it back, so Shiqi decided he wouldn't get any pie. But Shiqi was also forgetful Monday and might have given two or even all three pies to one person. How many different distributions of pie were there?

c) Tuesday, Shiqi had 8 identical pies. Shiqi didn't want anyone to lose out, so made sure everyone got at least one pie. Once everyone had a pie, anyone could come back for seconds. Or thirds! How many ways were there to distribute the pies on Tuesday?

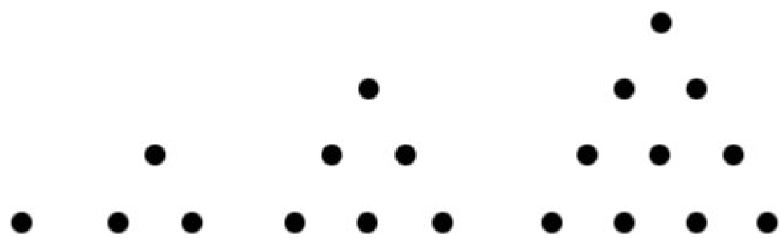
Creative Thinking

IMSA *Mu Alpha Theta*

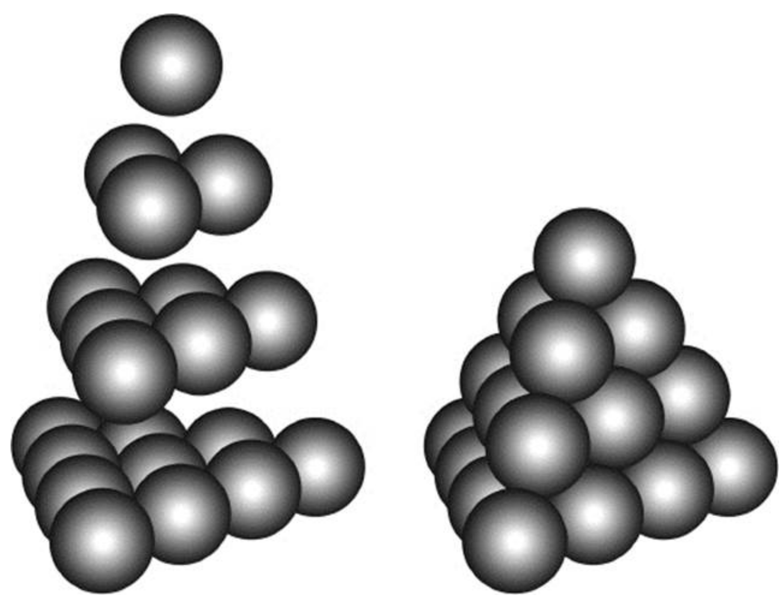
February 22, 2023

2 Triangles and Tetrahedra

A triangular number counts the number of dots arranged in a triangle. Thus, the picture below shows that the first four triangular numbers are $T_1 = 1$, $T_2 = 3$, $T_3 = 6$, and $T_4 = 10$.



In three-dimensions, we can stack balls into triangular pyramids, or *tetrahedra*. The diagram below shows how 20 balls can be stacked into a tetrahedron that has four balls on each edge, so $Te_4 = 20$.



a) If you are told that the 2023rd triangular number is $T_{2023} = 2{,}047{,}276$, compute T_{2024}.

b) If you are told that the 2023rd tetrahedral numbers is $Te_{2023} = 1{,}381{,}911{,}300$, compute Te_{2024}.

Creative Thinking

IMSA *Mu Alpha Theta*

February 22, 2023

3 Strange Operators

Two very strange operations on numbers are described below. Each operation takes two numbers and combines them, like plus or times does. Only these ones are a little stranger...

a) The operation $\diamond$ takes two numbers a and b and gives the result $a \diamond b$. It satisfies these rules: i) $a \diamond b = -(b \diamond a)$, and ii) $a \diamond (b + c) = (a \diamond b) + (a \diamond c)$. Use these rules to compute $2023 \diamond 2025$.

b) The even stranger operator $\circ$ takes two numbers a and b and gives the result $a \circ b$. It satisfies these rules: i) $a \circ \frac{1}{a} = -1$, and ii) $a \circ (b \cdot c) = (a \circ b) + (a \circ c)$. Compute the value of $(25 \circ 5) + (5 \circ 25)$.

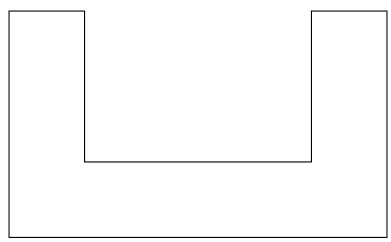
Creative Thinking

IMSA *Mu Alpha Theta*

February 22, 2023

4 Guarding the Art

Shown below is the floor plan of an art gallery.



You can count that there are 8 walls in this art gallery. It should be obvious that it takes at least two guards to watch the whole gallery at once, because a single guard can't be in a position to look into both "ends" at the same time.

a) Draw a floor plan for an art gallery with only 6 walls that still requires at least two guards. (Note: a "floor plan" is a polygon that shows the walls of the art gallery. Assume that "guard" can stand anywhere, and have 360° vision, but are not allowed to move from their position.)

b) What is the smallest number of walls in a gallery that requires at least three guards? Draw such a gallery.