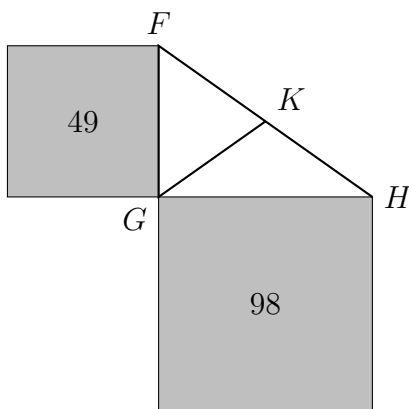


# 8<sup>th</sup> Grade Team Contest

IMSA *Mu Alpha Theta*

February 22, 2023

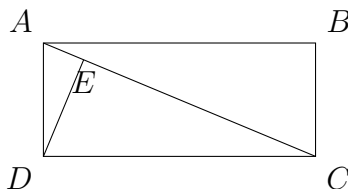
1. If 40% of  $x$  is 50, what is 50% of  $x$ ?
2. If 4 ducks eat 5 cupcakes in 3 minutes, how long does it take 9 ducks to eat 12 cupcakes? Answer in minutes and seconds, to the nearest whole second.
3. Consider the list of perfect squares: 1, 4, 9, 16, 25,  $\dots$ . The gap between the squares 9 and 16 is 7. There are two squares where the gap between them is 2023. What is the smaller of those two squares?
4. In the diagram below, the smaller square has an area of 49, and the larger square's area is 98. Segments  $\overline{FG}$  and  $\overline{GH}$  are perpendicular, and  $K$  is the midpoint of  $\overline{FH}$ . Compute the length  $GK$ .



5. Oren is stacking oranges for a display at the market. He is making a pyramid. He does this by creating a square with  $n \times n$  oranges in it. In the center of each square of four oranges in this bottom layer, he places another orange to create a layer that is a square with  $(n - 1) \times (n - 1)$  oranges in it. He continues upward, creating a  $(n - 2) \times (n - 2)$  layer on top of that, and so on until he places a single orange on the very top layer. For example, if his pyramid is 5 layers tall, it contains  $25 + 16 + 9 + 4 + 1 = 55$  oranges. What is the shortest pyramid that contains 2023 or more oranges?
6. Rohan is organizing chemicals for the lab. He is putting the chemicals into groups. He has three solids,  $A$ ,  $B$ , and  $C$ , and he has four liquids  $D$ ,  $E$ ,  $F$ , and  $G$ . No liquid may be placed in a group with a solid. Additionally,  $A$  and  $B$  cannot be in the same group

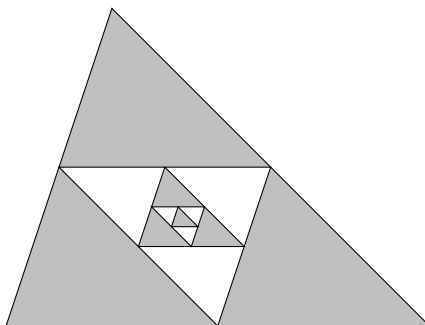
or they will create an explosive reaction, and  $D$  and  $G$  must also be separated or they will combine to create a poisonous cloud. How many different ways does Rohan have to group the chemicals that avoids all these problems?

7. A dartboard has a section that scores 10 points and a section that scores 7 points. Notice that you can never score a total of 15 points, because  $7 + 7 = 14$  is too small, but  $7 + 10 = 17$  and  $10 + 10 = 20$  are both too large of scores. What is the largest score that you cannot make?
8. Find the sum of all integers  $n$  so that  $\frac{n+5}{n-1}$  is also an integer.
9. The *Fibonacci numbers* are the sequence  $1, 1, 2, 3, 5, 8, \dots$ , where each number is the sum of the two previous numbers. To indicate where a number is within the sequence we use the notation  $F_1 = 1, F_2 = 1, F_3 = 2$ , etc. Find the exact value of the sum  $F_2 + F_4 + F_6 + \dots + F_{20} + F_{22}$ . Hint:  $F_{23} = 28656$ .
10. In the rectangle below,  $AB = 12$  and  $AC = 13$ . Additionally, segment  $\overline{DE}$  is perpendicular to  $\overline{AC}$ . Compute the exact area of triangle  $\triangle ADE$ .



11. How many positive integers up to 2023 have an odd prime number of positive divisors?
12. How many five-digit numbers have their digits strictly increasing from left to right?
13. A legendary tale says that Queen Dido was given “all the land she could fit within the skin of a bull” to found the ancient city of Carthage. She cleverly cut the skin into thin strips and laid them end-to-end to surround a large area with the sea as one side and thus founded quite a large city for herself! Your turn! If you have 400 meters of fencing, and you are to fence off a rectangular plot of land with a straight river as one side, what is the largest amount of area you can fence off?
14. You start by writing the number 30 eight times. Then you write the number 40 until the mean of all the numbers you have written becomes equal to 36. Then you write the number 32 until the median of all the numbers you have written to this point becomes 32. Finally, you write the number 45 until it becomes the mode. How many numbers have you written?
15. A new club has just formed at the library. Initially there are 35 boys and 15 girls in the club. But three times as many girls join the club as boys after that, so that by the end of the ratio of girls to boys is  $6/5$ . How many total girls and boys are in the club at the end of the year?

16. Find the sum of all integer values of  $y$  so that  $6y + 5$ ,  $20 - y$ , and 50 could be the three lengths of the sides of a triangle.
17. 17 competitors come to a math contest. Before the contest begins, each competitor shakes hands with each other competitor. How many handshakes occur?
18. Initially you have a triangle of area 12 that is painted blue. You connect the midpoints of its sides and paint the resulting triangle white. For that smaller triangle, you again connect the midpoints of its sides and paint the smaller triangle blue. You keep going, alternating painting blue and white, forever. At the end, what is the total amount of area that is painted blue?



19. Four  $A$ 's, four  $B$ 's, four  $C$ 's, and four  $D$ 's are to be placed in the grid below so that each row and each column contain exactly one copy of each letter. The top left square must contain an  $A$  and the bottom right must contain a  $D$ . How many possible arrangements of the letters are there that satisfy these conditions?

$A$			
			$D$

20. Mathsy makes an art piece by taking four identical semicircles and setting them in the corners of a square as shown. Each semicircle is flush up against the corner of the square, and is tangent to the semicircles in the neighboring corners. If the square has sides of length 1, how long is the segment shown that connects the centers of a pair of opposite semicircles?

