8^{th} Grade Individual Contest

IMSA *Mu Alpha Theta*

February 22, 2023

1. Compute $5 + 4 \cdot 3 - 2 \div 1$.

2. Connie has some coins. She has 6 quarters, 4 dimes, 3 nickels, and 7 pennies. Her sister Mona also has some money: 3 quarters, 2 dimes, 16 pennies, and a 50-cent piece. They give all their money to Nicole, who loves nickels. Nicole says "All this money is worth n nickels and three pennies." Compute the value of n.

3. Backwards Bob was told to increase all the prices in the store by 20%, but then to offer a 15% off sale. Naturally, he increased all the prices by 15% and then gave customers a 20% discount. How much are Bob's customers saving compared to what should have happened? Answer as a percentage.

4. Determine the value of x if $\dfrac{1}{1+\frac{1}{1+\frac{1}{x}}} = \dfrac{6}{5}$.

5. Old McDonald had a farm. And on this farm he had some cows and chickens. A cow has four legs and one head. A chicken has two legs and one head. On the far there are 60 legs and 24 heads. How many cows are there?

6. Find the sum of all the positive divisors of 2023.

7. A *palindrome* is a number like 484 or 2772 that reads the same way backward and forward. How many three-digit palindromes are there that are divisible by 15?

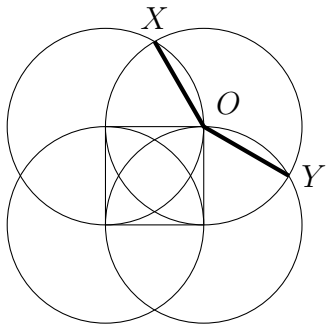
8. Rich used to have \$240. Then he discovered donuts. He bought and ate a lot of donuts. His weight increased from 150 pounds to 180 pounds. The amount of money he had decreased by the same percentage that his weight increased by. How much money does Rich have now?

9. Boca Buena Burritos was reviewing their Super Bowl party orders. They had 14 orders which averaged 23 burritos. No, wait! They just found one more order which increased the average to 25 burritos. How many burritos were part of that last order?

10. What is 50% of 40% of 30% of 20% of 10? Answer as a fraction in lowest terms.

11. A $3 \times 3 \times 5$ rectangular block is painted red on the outside. The block is then cut into $1 \times 1 \times 1$ cubes. Considering all these small cubes, compute the ratio of painted to unpainted sides. Answer as a reduced fraction.

12. Four circles, each with radius 1, are drawn with centers that are the four corners of a square, as shown. Find the measure of the angle $\angle XOY$ as shown in the diagram.



13. Define an operation on two numbers by $x \diamond y = \dfrac{x^2 - y^2}{x + y}$. If x can be any number from 2000 to 2023 and y can be any number from 1 to 10, how many different choices of x and y make $x \diamond y$ a whole number?

14. Lily is bored so she is taking a number, x, computing the values of $x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$, dividing that by 5, and writing down the remainder. If she ever gets up to $x = 2023$, what will be the remainder?

15. Salazar is creating a list of numbers. The first two numbers on his list are 1, 2. After that, the next number he puts on the list is the smallest whole number so that no three numbers on his list can be the lengths of sides of a triangle. For example, if 7 is on his list (hint: it is not) then 8 could not also go on the list, because there is a triangle whose sides have length 2, 7, and 8. What is the 10^{th} number on Salazar's list?

16. Yina writes the number 1 on the board and then starts a timer. At the end of each minute, she adds the number on the board to itself and adds 1, then writes the new number on the board in place of the old one. After how many minutes will 255 appear on the blackboard?

17. The inequality $1 + x < 5\sqrt{x}$ is true when $a < x < b$. Find $\sqrt{a} + \sqrt{b}$.

18. A cylinder holds five spheres. The cylinder and spheres each have radius 3 cm, so the spheres all touch the sides of the cylinder. The cylinder is just long enough to hold all the spheres end-to-end. How much volume inside the cylinder is not taken up by the spheres? (You may leave π in your answer.)

19. A collection of 7 numbers has average value x. If the largest number is removed, the average of the remaining numbers drops by 10%. If the smallest number is removed instead, the average of the remaining numbers increases by 15%. Compute the ratio (average of the middle five numbers)/x.

20. Charlie has many coins in his pockets. Charlie sees a chocolate bar he would like to buy that costs 40 cents. He starts pulling coins out of his pockets, getting a nickel, a dime, or a quarter with equal probability. What is the probability that he will have pulled out enough money to buy the candy bar within the first three coins he pulls out?