

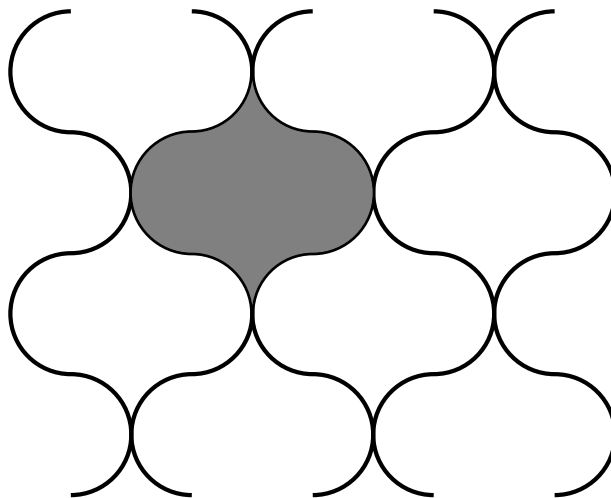
7th Grade Individual Contest

IMSA *Mu Alpha Theta*

February 22, 2023

1. Compute $73 - 7 - 3 - (7 - 3)$.
2. Rover sees a tasty bone 40 meters to the east and 30 meters to the south. He goes straight toward the bone and reaches it in 5 seconds. How fast, in meters per second, was Rover moving?
3. Solve for x : $\frac{5}{3} + 4 = \frac{x}{7} + 5$.
4. At the store, Rohan found a shirt he liked that cost \$15. It was on sale for 30% off, and he had a coupon that would give him 10% off of the sale price. To save time, the cashier just gave him 40% off the original price instead. How much money did Rohan save from the cashier's deal compared to if he had used the coupons without the cashier's deal?
5. Given that $y = 3x + 2$ and that $y = 2x + 6$, compute the numerical value of y .
6. Aashima took four tests in her math class last semester. Three of her scores were 85, 90, and 95. The average of her test scores was 91. What score did Aashima receive on her fourth test?
7. Find the number of positive integers less than 1001 that are not factors of 1001.
8. Two numbers, a and b , are chosen from 1 to 90. How many possible pairs (a, b) are there where $a + b$ is divisible by 12?
9. If n is a positive whole number, then $n!$ is the product of all the positive whole numbers up to, and including, n . For example, $5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$. Compute the greatest prime factor of $13! + 15!$.
10. The number x has four distinct positive divisors. The number y has five distinct positive divisors. The numbers x and y share no divisors other than 1. What is the smallest possible value of xy ?
11. Sandra is completing a very large problem set with 100 problems on it. Normally she can finish 12 problems in an hour. But after a while she gets tired and slows down to half that speed. She finishes the whole set in exactly 10 hours. How many problems had she completed before she got tired?

12. Find the sum of all A 's where A is one of the digits 1–9, and the four-digit number $AAA0$ is divisible by A^2 .
13. Compute the exact value of $\frac{2^{2023} + 2^{2022}}{2^{2021} + 2^{2020}}$.
14. Connie likes coins. She tosses a fair coin four times, and wins if she gets at least two heads. What is the probability that she wins?
15. The notation $|a|$ means the *absolute value* of a , which is a if $a \geq 0$ and $-a$ if $a < 0$. So $|4.3| = 4.3$, $|-12| = 12$ and $|0| = 0$. How many whole numbers satisfy the inequality $|x - 6| < 4$?
16. Just like multiplication is repeated addition ($3 \cdot 4 = 3 + 3 + 3 + 3$) and exponents are repeated multiplication ($3^4 = 3 \cdot 3 \cdot 3 \cdot 3$), the operation of *tetration* is repeated exponentiation. It is written as ${}^4 3 = 3^{3^{3^3}}$. Compute the units digit of ${}^3 4$.
17. Compute the largest integer that satisfies the inequality $\frac{4x + 15}{3} < 2$.
18. Triangle $\triangle ABC$ is isosceles, with sides $AB = AC$. Angle A has measure x in degrees. Angle B has measure $2x - y$ while and C has measure $3y + 8$. Determine the value of x .
19. Shiqi has found some wallpaper she really likes. The pattern is shown below, and is made entirely out of semicircles of diameter 1. In each column, the semicircles meet end-to-end and face opposite directions. Columns touch each other where the tangent lines to each semicircle are vertical. Compute the exact area of the shaded region.



20. The expression

$$\sqrt{4 + 2\sqrt{3}}$$

can be simplified to $a + \sqrt{b}$ where a and b are integers and b has no square factors. Compute $a + b$.