

8th Grade Team Contest

IMSA *Mu Alpha Theta*

March 2, 2022

1. Steven owns a farm in the countryside. On the farm, there are pigs and chickens and between these animals there are 24 heads and 80 legs. How many chickens are there?
2. A box contains red, green, and blue blocks. Of these blocks, 12 are not red, 7 are not green, and 17 are not blue. How many blocks are in the box?
3. Consider the expression $a \circ 2 \circ 2$. To form an equation, one of the \circ 's will be replaced with $=$ and the other will be replaced with $+$, $-$, \times , or \div . How many real numbers can be substituted for a such that there is a way of choosing symbols for the \circ 's to make a true equation?
4. Jeb can run m miles in 3 hours. At the same speed he would run 50 miles in $m - 5$ hours. At this speed, how long would it take Jeb to run 24 miles? Give your answer in *minutes*.

5. Solve for x :

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{x}}} = \frac{5}{4}$$

6. How many positive integers less than 2022 have an odd number of positive integer divisors?
7. A wealthy king has 15 equal chests full of gold coins. He decides to split his riches between his five children. To ensure none of the children will ever be poor, he decides to give each one at least two chests of gold. But the law of the kingdom is that the oldest child must get more than any other child. How many different ways are there to divide the chests among the children that follow these two conditions?
8. A large school is going to send all of its students to the zoo for a field trip. The ticket price is normally \$10 for adults and \$5.50 for students. But this week the zoo is having a special of 30% off on all student tickets. The school divides the students into equal sized groups, each with two adult chaperones. There are exactly 40 groups, and total cost turns out to be \$4958. How many students are in each group?
9. Tommy has some model train tracks. Each piece is identical to each other, and is a quarter circle. Obviously Tommy could take four of these pieces and make a circular track. How many pieces of track are needed to make the next shortest loop for his train? Note that trains can't make sudden turns, so the pieces of track must connect smoothly together.

10. When braiding you begin with three strands, one on the left, one in the middle, and one on the right. First, the middle strand is passed over the left strand. Now the former left strand, which is now in the middle, is passed over the right strand, and so the right strand becomes the new middle strand. These two steps are repeated as many times as desired.

If the strands are originally called A , B , and C from left to right, then after the first pass, the strands are now in the order BAC . Then after the second pass, they are in the order BCA . In what order will the strands be after exactly 55 steps?

11. The sum of seven positive integers is 42. The mean, mode, and median of these integers are consecutive integers (though not necessarily in that order). What is the largest possible difference between the largest and smallest of the seven integers?

12. Pentagon $ABCDE$ has sides of lengths $AB = 4$, $BC = 3$, $CD = 2$. Angles $\angle ABC$, $\angle ACD$, and $\angle AED$ are right angles, and side \overline{AE} is parallel to \overline{BC} . Find the area of $\triangle AED$.

13. Dr. Schrödinger put his cat in a box. In two minutes, the box will flip a coin, and if it flips heads the cat will be teleported to Australia. In that time, there is a 25% chance that the cat will chew through the wires and deactivate the teleporter. After two minutes, the doctor looks into the box and finds that the cat was not teleported (meaning either it chewed through the wires, the box flipped tails, or both). Given that the cat was not teleported, what is the probability that the cat chewed through the wires?

14. A polyhedron is a three-dimensional shape all of whose sides are polygons. If a polyhedron has no holes that go all the way through it, then it satisfies an amazing formula called *Euler's formula* which says that $V + F - E = 2$, where V is the number of vertices (corners) on the shape, F is the number of faces (sides), and E is the number of edges where two faces meet. For instance, a cube has six faces, eight vertices, and twelve edges, and indeed $6 + 8 - 12 = 2$.

Consider a polyhedron all of whose faces are pentagons, and three pentagons fit together around every vertex. Determine the number of edges possessed by this polyhedron.

15. You wish to dig a deep hole to plant a big tree. Your neighbor Anne is a professional hole digger, and can dig your hole in 4 hours. Your other neighbor Beth has a shovel-o-matic which can dig your hole in only 2 hours. Your dog also likes to dig holes, and could dig your hole in 8 hours, but there is a 75% chance that a squirrel will run by and your dog will chase the squirrel instead of working on the hole. If you decide to use all three, how long to you expect it will take to dig the hole?
16. Find the smallest positive odd number where the sum of its divisors is at least twice the number itself. For instance, the divisors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30, which sum to 72, which is more than twice 30. Find the smallest odd number for which this is true.