

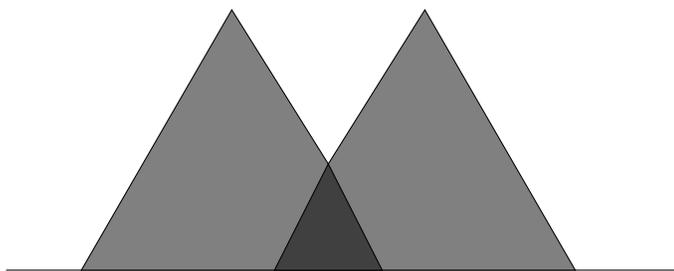
8th Grade Individual Contest

IMSA *Mu Alpha Theta*

March 2, 2022

1. The expression $5^5 4^4 - 4^5 4^5 - 4^4 5^5$ can be written in the form $-a^b$ where a is prime and b is an integer. Compute $a + b$.
2. Determine the number positive integers less than 100 whose ones digit is the same as that of their square.
3. You are given 100 logs, each of which can be sawn into either two planks or into two rods. Given that four rods and one plank are required to construct a table, determine the maximum number of tables that could be built with the 100 logs.
4. 100 people are waiting in line. The first person has \$1 in their pocket, the second has \$2, the third has \$3, and so on. Mary walks by the line twice. The first time she takes half the money of every person whose place in line is an even number. The second time she takes all the money of every person whose place in line is a perfect square. How much money does Mary have in the end?
5. Rohan has to read a 600-page book for his final in 10-hours. He reads at a certain rate for 4 hours and then takes a nap for 3 hours. When he wakes up he realizes that he must now read at twice the earlier rate if he wants to finish before his final. How many pages per hour must Rohan read after his nap if he wants to finish on time?
6. The city of Wamalone is having their annual Watermelon Feast and has brought plenty of watermelons for everyone. Everyone gets their turn to choose their own watermelons. First, Wallace takes $\frac{1}{3}$ of the watermelons and starts eating them. Then Wendy grabs $\frac{1}{5}$ of the remaining watermelons. Their first child, Wesley, takes $\frac{1}{4}$ of the remaining watermelons. Winnie then takes $\frac{1}{2}$ of the remaining watermelons, and lastly, Walter takes $\frac{1}{6}$ of the remaining watermelons, which left 10 watermelons for later. How many watermelons were there to begin with?
7. Suzi's address is made up of only prime digits, has no digits that are repeated three or more times, and is divisible by 3. What's the largest 4-digit number that her address could be?
8. Compute the number of pairs (x, y) where x and y are positive integers between 1 and 25 inclusive, and where $x^2 - y^2$ is evenly divisible by 5.

9. Shiqi wishes to cut a $3 \times 3 \times 3$ cube into twenty-seven $1 \times 1 \times 1$ unit cubes. What is the minimum amount of cuts Shiqi will have to make if the pieces can be rearranged and stacked on top of each other after each cut?
10. Carl plants a mushroom at noon on Day 1. At noon on each subsequent day any mushroom that has been alive for two or more days causes a new baby mushroom to sprout. On what number day is there first more than 100 mushrooms total?
11. The six faces of two fair die are labeled $-3, -2, -1, 0, 1,$ and 2 . Suppose Yina rolls these two dice and adds the two numbers that come up. What is the probability that her sum is non-negative?
12. Two equilateral triangles have sides of length 4 and overlap each other in a smaller equilateral triangle as shown in the diagram below.



The overlapping region, shown in a darker tone in the diagram above, has area exactly $1/11$ of the total enclosed area (all the shaded area, counting the darker part only once). Compute the altitude of the smaller triangle.

13. Eric has a 2 inch by 2 inch square grid that is divided into four 1 inch by 1 inch squares. For each of the 1×1 squares, Eric flips a coin, and if the coin comes up heads (exactly 50% chance), he colors in that square. After repeating this procedure for each of the four 1×1 squares, compute the probability that the colored area has a perimeter of exactly 8 inches.
14. Given that $x^2 + 3 = 0$, compute $x^4 + 7x^2 + 19$.
15. What is the remainder when $(4^7 + 5^7)^6$ is divided by 7?
16. A coin rolls around the inside of a circular track with a radius that is $\frac{8}{3}$ times the coin's radius. The coin starts at the bottom of the track with an arrow on its face that is pointing upward. The coin rolls around the track until it returns to its original position and orientation—at the bottom with the arrow pointing upward. How many rotations did the coin perform (one rotation corresponds to the arrow spinning once in a circle)?