

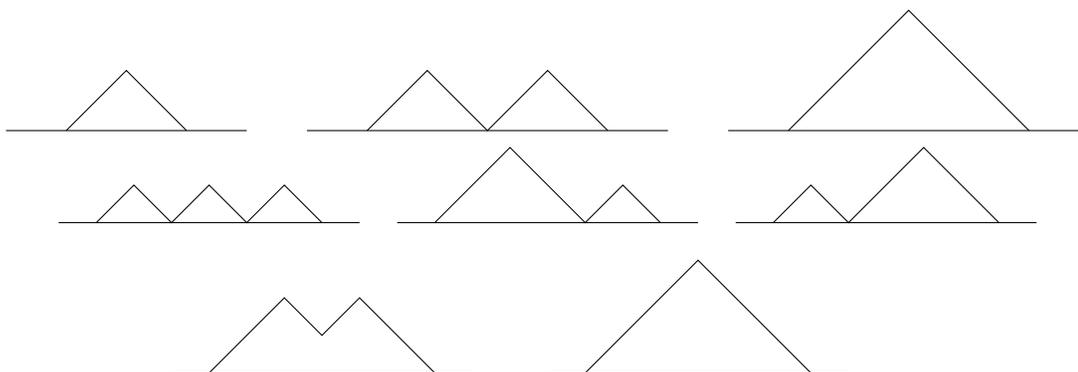
7th Grade Team Contest

IMSA *Mu Alpha Theta*

March 2, 2022

1. At the Illinois Music and Arts Academy there are 100 students each of whom must take at least one art class. If you know that 50 students take drawing, 30 students take painting, 40 students take sculpting, and 5 students take all three, how many students take exactly two art classes?
2. Find the number of possible digits B which make the three-digit integer $4B2$ a multiple of 4.
3. A bus is exactly filled to capacity with passengers. At the first stop, half of the people plus ten more get off the bus. At the second stop, 20 people get on the bus. At the third stop, $\frac{1}{3}$ of the people on the bus get off. At the fourth stop, 5 people get on the bus. The bus is now exactly 50% full. What is the capacity of the bus?
4. There are 3 blue marbles, 3 red marbles, and 3 yellow marbles. Each pair of differently colored marbles is connected by a string. How many strings are there?
5. A super duper bouncy ball triples its bounce height, minus one foot, on each successive bounce. For example, if it were to be dropped from 2 feet, it would reach a height of $3 \times 2 - 1 = 5$ feet after the first bounce, a height of $3 \times 5 - 1 = 14$ feet after the second, and so forth. If the ball is dropped from a height of 4 feet, after how many bounces does the balls height first surpass 1000 feet?
6. Mike has a bag full of toy dinosaurs, which are yellow, blue, green and red. A third of the dinosaurs are yellow, a third of the number of yellow dinosaurs is half the number of blue dinosaurs, and the number of yellow dinosaurs and blue dinosaurs combined are double the number of green dinosaurs. In simplest terms, what fraction of Mike's dinosaurs are red?
7. The length of a rectangle is 4 times its width. When both dimensions are increased by 2, the area increases by 24. What is the perimeter of the original rectangle?
8. In how many ways can the letters of the word "RETREAT" be rearranged so that the resulting string of letters is the same when read forwards or backwards? Note that the rearrangements are not required to be actual words.
9. Find the sum of the coefficients in the expansion of $(2x - 7)^5$

10. The greatest integer less than or equal to x is a . If $2x + 3a = 45.2$, what is the value of $10x$?
11. A special team is chosen from a group of six candidates: Abby, Bobby, Carl, Danielle, Evan, and Faith. If at least one of Carl and Faith made the team, and Abby cannot be on the team with Carl, how many distinct teams could have been chosen? (A team can consist of just one person.)
12. A farmer tries to divide his cows into groups of 2, but finds he has one left over. He does the same for groups of 3, and 5, but finds that he has one left over each time. When he tries again with groups of 11, he is finally able to divide his cows evenly. What is the smallest number of cows the farmer could own?
13. Billy was adding up a column of one-, two-, and three-digit numbers. However, he accidentally write the digits of one of the numbers in the wrong order. For example, he might have written 85 as 58 or 233 and 323. So his answer is wrong. Determine the least possible positive difference between the correct sum and the sum Billy calculated.
14. The line with equation $x = y - 5$ and the parabola with equation $y = 2x^2 + x - 3$ intersect in two points in the plane. Compute the distance between these points.
15. Rob the robot is on an infinite checkerboard. When Rob is on a black square, he will either move 4 squares up, 2 squares left, 3 squares right, or 2 squares down all with equal probability. When Rob is on a white square, he will either move 2 squares up, 6 squares left, or 2 squares right, all with equal probability. If Rob starts on a black square, what's the highest number of moves after which he is still most likely to be on a black square?
16. We can draw a *mountain range of width n* by starting at ground level, and taking n up-steps and n down-steps, never going below ground level. There is one mountain range of width 1, two of width 2, and 5 of width 3 as shown below.



Determine the number of mountain ranges of width 5.