Creative Thinking

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1 Do you speak Tralfamadorian?

On the far away planet of Tralfamador, the natives write in a very weird language called Tralfamadorian. Here is how they form sentences. First, they start with a single symbol S. Then, they use the rules below to replace characters with others until they get a sentence which contains no symbols other than ■, ▲, ★, or ♦:

Symbol	Gets Replaced By	Symbols
S	$\rightarrow$	NVO
N	$\rightarrow$	★▲ or ★ON or ▲ON
V	$\rightarrow$	♦★ or ♦OV
O	$\rightarrow$	■ or ♦VO

For example, ★▲♦■♦★■ is a possible sentence in Tralfamadorian since we can apply the rules as follows:

1. $S \rightarrow NVO$
2. $NVO \rightarrow$ ★▲VO (replacing the N)
3. ★▲$VO \rightarrow$ ★▲♦OVO (replacing the V)
4. ★▲♦$OVO \rightarrow$ ★▲♦■V■ (*two* replacements of O)
5. ★▲♦■V■ $\rightarrow$ ★▲♦■♦★■ (replacing the V)

In this case, the rules were applied 6 times–note the double application in step 4 counts twice.

The Problems

1. The conbination ★■▲■★▲♦■♦★♦♦★■ is a legitimate Tralfamadorian sentence. Determine the number of rule applications (in the sense that the example above took 6 applications) to derive this sentence.
2. Determine the number of possible sentences in Tralfamadorian that contain nine or fewer characters.

2 Aztec Diamonds

Atzi really likes assembling Aztec diamonds. An *Aztec diamond of order* n is a shape made out of four "staircases" with n steps glued together back-to-back. See several Aztec diamonds below:

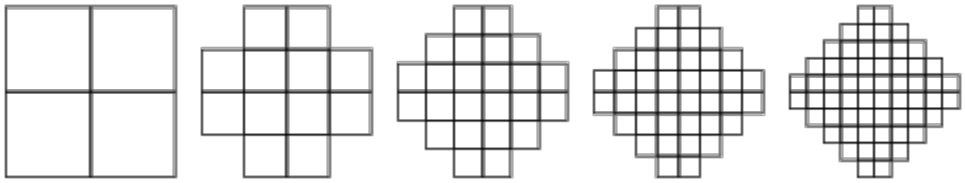

Figure 1: (from left to right) Aztec Diamonds of order $1, 2, 3, 4, 5$, respectively.

A *pentomino* is a shape made from five squares glued together. There are 12 of these shapes:

Figure 2: All 12 possible pentominoes.

Rotating a pentomino or turning it over to get its mirror image is allowed

The Problems

1. Atzi has a bag with a lot of pentominoes of each possible shape. Which of the Aztec diamonds in Figure 1 can Atzi assemble out of the pentominoes? Remember that they can be rotated or flipped if necessary.

2. Find the greatest number $n < 100$ for which Atzi can assemble the Aztec diamond of order n out of pentominoes by the same rules as in the previous problem.

3. Atzi also has a huge box of identical dominoes, such as the ones below. In how many unique ways can Atzi assemble an Aztec diamond of order 3 with the dominoes? As before, the dominoes can be vertical or horizontal, as needed.

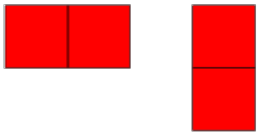

Figure 3: A Domino in its horizontal and vertical orientations.