

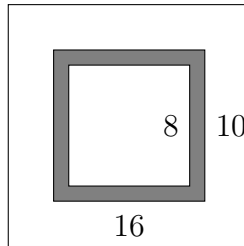
8th Grade Individual Contest

IMSA *Mu Alpha Theta*

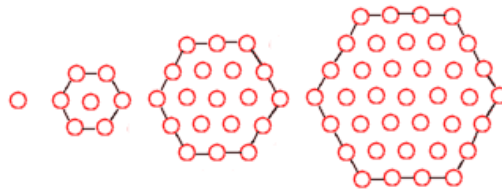
March 10, 2021

1. The sum of five consecutive integers is 145. Determine the sum of the largest and smallest of these five.
2. A store sells a product at a profit of 20%. That is, the selling price is 20% more than the price the store paid to buy the items. When sales become slow, the store drops the price of those items by 15% off (of that higher price that they were selling them for). The items are now selling at a profit of $x\%$ of what the store paid to buy the items. Determine x .
3. For how many positive integers n is \sqrt{n} a distance of at most 1 from 10?
4. Billy and Bob ride their bikes from school to the library, starting at the same time, along the same route. The library is 36 miles away from the school. Billy travels 6 miles per hour slower than Bob. Bob gets to the library, then turns back to ride back to Billy, and they meet 15 miles from the library. How fast was Billy traveling (in miles per hour)?
5. A sequence of numbers is called an *arithmetic sequence* if the difference between each pair of consecutive terms is always the same, such as 8, 5, 2, $-1, \dots$, with the difference -3 each time. Given that 16, $a, b, c, d, e, 58$ is an arithmetic sequence, compute the value of d .
6. A square whose sides have length 4 has one corner at the origin, one to $(0, 4)$, and one at $(4, 0)$. The square rotates around its vertex at $(4, 0)$, rolling over one quarter-turn (so that the vertex that was at $(4, 4)$ is now at $(8, 0)$). During this roll, the center of the square traces out an arc in the coordinate plane. Determine the area between this arc and the x -axis.
7. For a magazine story, the writer was paid \$100 plus fifty cents per word. The photographer was paid \$150 per photograph. The editor demanded that there was one picture for every 200 words. The writer and photographer were paid the same amount. How many words were in the story?
8. Compute the length of the portion of the line $5y = -12x + 60$ that lies in the first quadrant.
9. Determine the rightmost two digits of 41^{3589} .

10. A white square with sides of length 8 cm is placed at the center of a shaded square whose sides have length 10 cm. This is, in turn, placed in the center of a white square with sides of length 16 cm, as shown below. A dart is thrown at this target and sticks in a random location within the largest square. What is the probability that the dart has stuck in the shaded portion of the target?



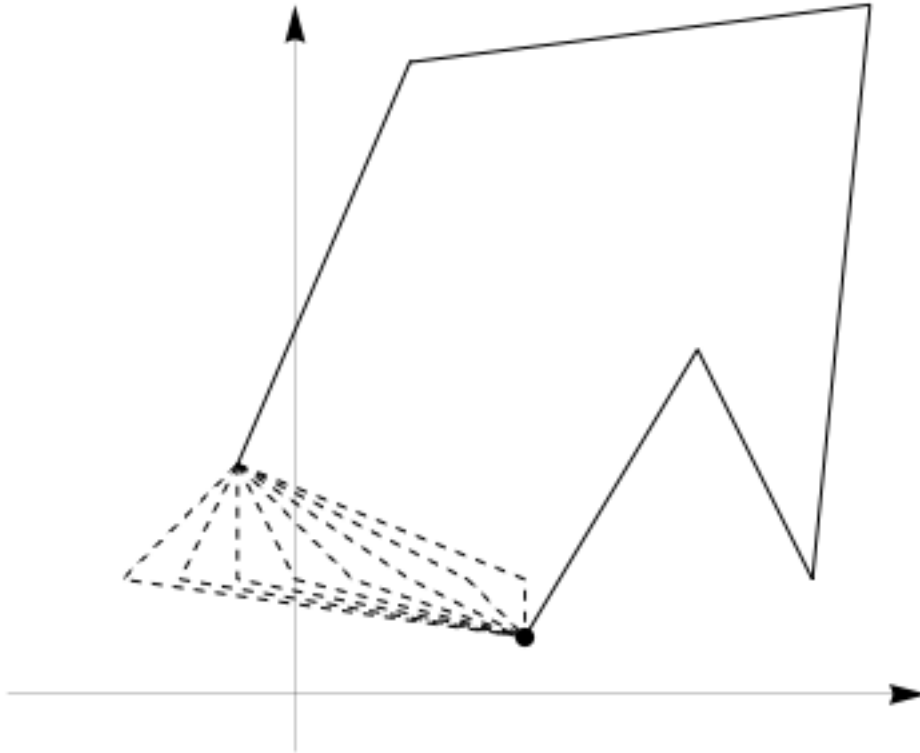
11. What is the smallest number larger than 4 that is a palindrome when expressed in both base 3 and base 5? (Express your answer in regular base 10).
12. There are 25 people in a class. 7 of them play badminton, and 13 of them play tennis. 4 people play both badminton and tennis. If two students are selected at random, what is the probability that neither of them plays either sport?
13. Eric draws a sequence of hexagons made up of dots on a piece of paper, as shown below:



The first "hexagon" he draws is made up of only one dot, the second is made of 7, third has 19 dots, the fourth has 37. If Eric continues to draw hexagons in this pattern, how many dots would the tenth hexagon have?

14. A $4 \times 6 \times h$ box and a $3 \times 5 \times 2h$ box have the same surface area. Determine h .
15. Rohan rolls a four-sided die (whose sides are numbered 1–4) and six-sided die (with sides numbered 1–6). What is the probability that the six-sider rolls a higher number than the four-sider?
16. A regular polygon has internal angles of 156° . How many diagonals does it have?
17. If n is a positive integer, then $n!$ means $1 \cdot 2 \cdot 3 \cdots n$. Determine the units digit of $(1!)^{10} + (2!)^9 + (3!)^8 + \cdots + (9!)^2 + (10!)^1$.
18. Two different numbers are selected from the set $\{1, 2, 3, 4, 5, 6\}$. Determine the probability that the product of the two numbers is divisible by 4
19. Starting at the point indicated and going around in the counterclockwise direction, the coordinates of the polygon shown below are $(4, 1)$, $(7, 6)$, $(9, 2)$, $(10, 12)$, $(2, 11)$, $(-1, 4)$,

and $(x, 2)$, this last point being free to slide back and forth as indicated in the diagram. If the area of the polygon is exactly 90, find x .



20. As shown below, three circles of radii 3, 1, and 2 (in that order) lie tangent to each other with collinear centers. A fourth circle (shaded in the diagram) is tangent to all three, as shown. Determine the radius of this fourth circle.

