7^{th} Grade Team Contest

IMSA *Mu Alpha Theta*

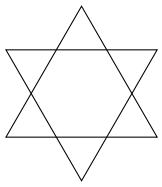
March 10, 2021

1. Find the six-digit number that fits the following description: All of its digits are different. The hundred-thousands digit is a factor of 25, the ten-thousands digit is a number that is neither prime nor composite, the thousands digits is the sum of the previous two digits. The hundreds digit is the sum of ten thousands digit and the thousands digit, the tens digit is 4 times the value of the ones digit, and the ones digit is an even prime number.

2. The set of positive integers $\{4, 4, 5, 6, x, y, z\}$ has an average value of 4. How many different combinations of x, y, and z are possible?

3. Compute the mean of all the integers from 1 to 25, inclusive.

4. Two equilateral triangles, with common center but pointing in opposite directions, overlap to form a six-pointed star, as shown:



   The area of the hexagon common to both triangles is 60. Determine the area of one of the original triangles.

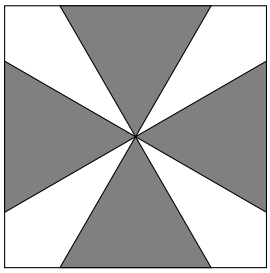
5. A restaurant offers three kinds of desserts, and exactly twice as many different kinds of appetizers as main courses. A dinner combination consists of an appetizer, a main course, and a dessert. What is the least number of main courses that the restaurant can offer so that a customer could have a different dinner combination each night in the year 2021?

6. When organizing her toys, Shea notices that if she sorts them into groups of 3, 4, 5, or 6 there is always one toy left over. If Shea has more than 100 toys but less than 150, how many toys does she have?

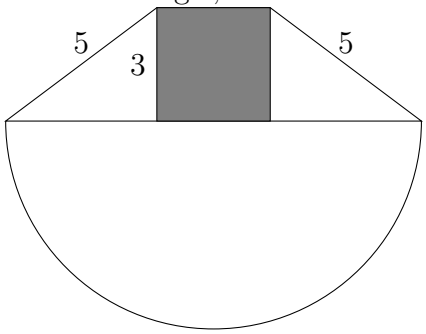
7. In MathNite, 2 pows are worth the same as a bop, 6 bops are worth the same as a K.O., and 9 gops are equivalent to a K.O. By how many percent is a gop is greater than a pow? Round your answer to the nearest whole percent.

8. The number 29A38B (where A and B are individual digits) is divisible by 9. How many such numbers are there?

9. The figure below shows four congruent equilateral triangles inside a square. If the area of each triangle is 2, compute the exact area of the square.

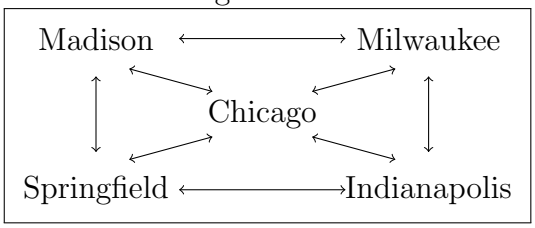


10. In the figure below, the bottom portion is a semicircle whose area is 15.125π. Some of the segments in the top portion of the figure have lengths as shown in the diagram. Given that the shaded region is a rectangle, determine its area.



11. Two ordinary fair dice are rolled. Determine the probability that the total shown on the dice is a square number.

12. A small local airline flies between Chicago, Madison, Milwaukee, Indianapolis, and Springfield. There is no direct flight between Madison to Indianapolis or between Milwaukee and Springfield. So all direct flights are as shown in the diagram below:



Compute the number of different routes a plane may take from Madison to Indianapolis, taking off at most three times.

13. How many palindromes are there between 20,000 and 30,000 that are divisible by 6?

14. Think of a Rubik's cube as made of only the smaller pieces you can see, with stickers on those pieces to make the sides different colors. A typical 3×3 Rubik's cube has 26 pieces (the center cannot be seen) and 54 stickers (nine of each of the six sides). For a 10×10 cube, compute the ratio of pieces to stickers, giving your answer as a reduced fraction.

15. The number $10^{2021} - 2021$ is written out fully in decimal form. Determine the sum of its digits.

16. Determine the sum of the digits of $111{,}111{,}111^2$.

17. What is the probability that a randomly chosen positive divisor of 660 is even?

18. At a certain school, grades are given on a curve. The top 15% of students earn an A, the top 40% earn a B or higher (since the top 40% will include the top 15%!) and the top 65% each a C or higher. On the latest math test, none of the 40 students in the school earned the same score. One student who scored 29 points earned a C grade. What is the lowest possible score on the test that might earn an A grade?

19. In Catdice, the lions roll an 8-sided die and the tigers roll a 10-sided die. The lions win if their roll is a 7 or 8 and the tigers roll 7 or less. The tigers win if they roll 8 or more and the lions roll 6 or less. Otherwise that round is a tie and more rounds are played until one team wins. What is the probability that the tigers win? State your answer as a fraction in lowest terms.

20. In the game described in the previous problem, if the lions and tigers play a large number of games, compute the average number of round these games last before someone wins.